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A Hydro-Magnetic Fluid Flow past a Rotating Semi-infinite Vertical Plate in the Presence of Variable Inclined Magnetic Field

S. N. Muthiga, M. N. Kinyanjui, P.R. Kiogora

Abstract: ***Stokes problem for a free convective flow past a vertical semi-infinite plate in a rotating system field taking into account the effect of viscous dissipation and joule heating in the presence of variable inclined magnetic field. An induced electric current known as Hall current exists due to the presence of both electric field and magnetic field. The fluid is subjected to a variable magnetic field inclined at an angle α with positive direction of x axis in the xz- plane. The central finite difference is used to discretisize space variables and Gauss Siedel iteration is used to advance time variable. The aim of the present investigation is to study the effects of variable inclined magnetic field on the velocity and Temperature profiles. Further the effect of angle of inclination, Prandtl number and Magnetic Reynolds number on the flow variables has been investigated. The effects of external cooling (Gr>0) of the plate by the free convection are studied. The skin friction and the rate of heat transfer are calculated using Newton's interpolation formula. The results obtained here are useful in applications on heat exchanger designs, wire and glass fiber drawing and in nuclear engineering in connection with the cooling of reactors.***

Index Terms: **Free convection, Hall Currents, Magnetic field, Rotation, Skin friction, Heat transfer.**

I. INTRODUCTION

The theoretical study of MHD flows has been a subject of great interest due to its widely application on designing of cooling systems with liquid metals, petroleum industry, purification of crude oil, separation of matter from fluids and many other applications. Extensive work has been published on flow past a vertical plate under different conditions. The analytical method fails to solve the problem of unsteady two-dimensional natural convection in a rotating system past a vertical semi-infinite plate considering joule heating in the presence of inclined magnetic field. The advent of advanced numerical methods and the developments in computer technology pave the way to solve such difficult problems. Finite difference methods play an important role in solving the partial differential equations. Hydro magnetic free convection flow past a semi-infinite vertical porous plate subjected to constant Heat Flux with Radiation Absorption was studied by Kwanza*et.al.* [1].Soundalgekar, Singh and Takhar [2] studied MHD free convection flow past a semi-infinite vertical plate with suction and injection. Viscous dissipation and joule heating effects on MHD free convection from a vertical plate with power-law variation in surface temperature in the presence of Hall and ion-slip currents by [3].Murali *et. al.* [4] investigated unsteady Magneto hydro dynamics free flow past a vertical porous plate. Hall Currents effects on free convection MHD flow past a porous plate was investigated by Satya *et. al.* [5].Unsteady MHD free convection heat and mass transfer flow past a semi-infinite vertical permeable moving plate with heat Absorption, Radiation, chemical Reaction and Soret Effects was studied by Madhusudhana*et. al*. [6].Gaurav Kumar Sharma *et. al*. [7] studied unsteady flow through porous media past a moving vertical plate with variable temperature in the presence of inclined magnetic field. Marigi*et. al*. [8] investigated Hydro magnetic Turbulent flow of a Rotating system past a semi-infinite vertical plate with Hall current. Sato [9] studied Hall effects on unsteady MHD free convection flow past an impulsively started porous plate with viscous and joule dissipation. Effect of inclined magnetic field on unsteady free convection flow of dissipative fluid past a vertical plate was studied by Sandeep *et. al.* [10]. Rajasekhar *et. al*. [11] investigated unsteady MHD free convection flow past a semi-infinite vertical porous plate. Palani and Srikanth [12] studied MHD flow past a semi-infinite vertical plate with mass transfer. Inspite of all these studies, much has not been done on a convective flow past a semi-infinite vertical plate in a rotating system with viscous dissipation and Joule Heating in the presence of variable inclined magnetic field.

II. MATHEMATICAL ANALYSIS

Consider unsteady, laminar, incompressible, free convection boundary layer flow of an electrically conducting fluid past a Rotating Semi-infinite vertical plate with joule heating in the presence of variable inclined magnetic field .The initial temperature of the fluid is the same as that of the fluid, but at time t>0 the plate starts moving

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impulsively in its own with plane with a constant velocity u_0 and its temperature instantaneously rises or falls to T_w which thereafter is maintained as such. The fluid is assumed to have constant properties except that the influence of the density variations with the temperature, following the well-known Boussinesq approximation [13] is considered only in the body force terms. The x-axis is taken along the semi-infinite vertical porous plate in the upward direction and z-axis normal to the wall .A variable inclined magnetic field is applied in x z axis at an angle α in the positive quadrant. Since the effects of Hall current give rise to a force in y direction which induces a cross flow in that direction, the flow becomes three dimensional.

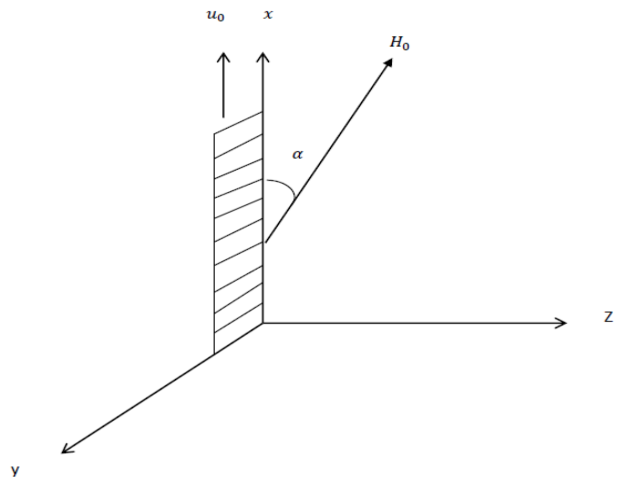

Fig 1: Flow Configuration

To simplify the problem, we assume that there is no variation of flow and temperature in the y directions. We let the fluid and the plate to be in a state of rigid rotation with uniform angular velocity Ω about the z-axis taken normal to the plate. Following Cramer and Pai [10] and Shercliff [11] we take the following vectorial field equations,

$$\mathbf{q}=(u,v,0)\ \mathbf{H}=(H_x+H_0\cos\theta,0,H_z+H_0\sin\theta),\mathbf{E}=(E_x,E_y,E_z)\text{ and }\mathbf{J}=(J_x,J_y,J_z) \quad (1)$$

Ohm's law for a moving conductor incorporating Hall current takes the form:

$$\mathbf{J}+\frac{\omega_e\tau_e}{\mathbf{H}_0}[\mathbf{J}\times\mathbf{H}]=\sigma\left[\mathbf{E}+\mu_e\mathbf{q}\times\mathbf{H}+\frac{1}{en_e}\nabla\cdot p_e\right] \quad (2)$$

where ω_e is the cyclotron frequency, τ_e the collision time, σ the electrical conductivity , μ_e the electrical conductivity p_e the electron pressure and η_e is the number density of electron. It has been assumed that ion-slip and thermoelectric effect is negligible. Further it is considered that electric field E=0 (Meyer, 1958) and electron pressure have been neglected. The equation of conservation of electric charge $\nabla.\mathbf{j}=\mathbf{j}_z$ =constant. This constant is zero since $\mathbf{j}_z=0$ which is electrically non-conducting.Thus $\mathbf{j}_z=0$ everywhere in the flow. Under this assumption equation (2) in components form becomes

$$J_x=\frac{\sigma\mu_e H_0(H_z+H_0\sin\alpha)\big(m(H_z+H_0\sin\alpha)u+H_0v\big)}{H_0^2+m^2(H_z+H_0\sin\alpha)^2} \quad (3)$$

$$J_y=\frac{\sigma\mu_e H_0(H_z+H_0\sin\alpha)\big(m(H_z+H_0\sin\alpha)v-H_0u\big)}{H_0^2+m^2(H_z+H_0\sin\alpha)^2} \quad (4)$$

In a rotating frame the governing boundary layer equations of mass, momentum and energy for free convection flows with the Boussinesq approximation are as follows

The Continuity Equation:

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \qquad (5)$$

The Momentum Equations:

$$\frac{\partial u}{\partial t}-u_0\frac{\partial u}{\partial z}+u\frac{\partial u}{\partial x}-2\Omega v=g\beta\left(T-T_\infty\right)+\upsilon\left(\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 u}{\partial x^2}\right)+\frac{\sigma\mu_e\left(H_z+H_0\sin\alpha\right)^2\left(m\left(H_z+H_0\sin\alpha\right)v-H_0u\right)}{\rho\left(H_0^{\ 2}+m^2\left(H_z+H_0\sin\alpha\right)^2\right)} \qquad (6)$$

$$\frac{\partial v}{\partial t}-u_0\frac{\partial v}{\partial z}+u\frac{\partial v}{\partial x}+2\Omega u=+\upsilon\left(\frac{\partial^2 v}{\partial z^2}+\frac{\partial^2 v}{\partial x^2}\right)+\frac{\sigma\mu_e\left(H_z+H_0\sin\alpha\right)^2\left(m\left(H_z+H_0\sin\alpha\right)u+H_0v\right)}{\rho\left(H_0^{\ 2}+m^2\left(H_z+H_0\sin\alpha\right)^2\right)} \qquad (7)$$

The Energy Equation:

$$\rho c_p\left[\frac{\partial T}{\partial t}+u_0\frac{\partial T}{\partial z}+u\frac{\partial T}{\partial x}\right]=k\left[\frac{\partial^2 T}{\partial z^2}+\frac{\partial^2 T}{\partial x^2}\right]+\mu\left[\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2\right]+\sigma\mu^2{}_e\left(H_z+H_0\sin\alpha\right)^2\left(v^2+u^2\right) \qquad (8)$$

$$\frac{\partial H_x}{\partial t}=H_0\sin\alpha\frac{\partial u}{\partial z}+\frac{\partial}{\partial z}\left(uH_z\right)+\frac{1}{\sigma\mu_e}\left(\frac{\partial^2 H_x}{\partial x^2}+\frac{\partial^2 H_x}{\partial z^2}\right) \qquad (9)$$

$$\frac{\partial H_z}{\partial t}=H_0\sin\alpha\frac{\partial u}{\partial z}-\frac{\partial}{\partial x}\left(uH_z\right)+\frac{1}{\sigma\mu_e}\left(\frac{\partial^2 H_z}{\partial x^2}+\frac{\partial^2 H_z}{\partial z^2}\right)) \qquad (10)$$

with corresponding boundary conditions:

$$\begin{aligned}
&t\le 0 \quad u\left(x,z,t\right)=0,\quad v\left(x,z,t\right)=0,\quad T\left(x,z,t\right)=0\\
&t>0 \quad u\left(x,z,t\right)=u_0,\ v\left(0,z,t\right)=0 \quad T\left(0,z,t\right)=T_w\\
&\qquad\ \ u\left(\infty,z,t\right)=0,\ \ v\left(\infty,z,t\right)=0 \quad T\left(\infty,z,t\right)=0
\end{aligned} \qquad (11)$$

Where u,v and w are the x,y and z components of velocity vector respectively.To obtain the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced :

$$u^*=\frac{u}{u_0},\qquad v^*=\frac{v}{u_0},\qquad t^*=\frac{u_0^{\ 2}t}{\upsilon}$$

$$x^*=\frac{xu_0}{\upsilon},\qquad z^*=\frac{zu_0}{\upsilon},\qquad H_x^{\ *}=\frac{H_x}{H_0},\qquad H_z^{\ *}=\frac{H_z}{H_0},\qquad \theta=\frac{T-T_\infty}{T_w-T_\infty} \qquad (12)$$

Substituting (12), equations (5),(6) , (7) , (8),(9) and (10) becomes

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial w^*}{\partial z^*}=0 \qquad (13)$$

$$\frac{\partial u^*}{\partial t^*}-u_0\frac{\partial u^*}{\partial z^*}+u^*\frac{\partial u^*}{\partial x^*}-2Erv=Gr\theta+\left(\frac{\partial^2 u^*}{\partial z^{*2}}+\frac{\partial^2 u^*}{\partial x^{*2}}\right)+M\left(H^*{}_z+\sin\alpha\right)^2\left[\frac{m\left(H^*{}_z+\sin\alpha\right)v^*-u^*}{1+m^2(H^*{}_z+\sin\alpha)^2}\right] \qquad (14)$$

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$$\frac{\partial v^*}{\partial t^*}-u_0\frac{\partial v^*}{\partial z^*}+u^*\frac{\partial v^*}{\partial x^*}+2Eru=\left(\frac{\partial^2 v^*}{\partial z^{*2}}+\frac{\partial^2 v^*}{\partial z^{*2}}\right)-M\left(H^*_z+\sin\alpha\right)^2\left[\frac{m\left(H^*_z+\sin\alpha\right)u^*+v^*}{1+m^2(H^*_z+\sin\alpha)^2}\right] \tag{15}$$

$$\frac{\partial \theta}{\partial t^*}-u_0\frac{\partial \theta}{\partial z^*}+u^*\frac{\partial \theta}{\partial x^*}=\frac{1}{\Pr}\left(\frac{\partial^2\theta}{\partial z^{*2}}+\frac{\partial^2\theta}{\partial x^{*2}}\right)+Ec\left(\left(\frac{\partial u^*}{\partial z^*}\right)^2+\left(\frac{\partial v^*}{\partial z^*}\right)^2\right)+R\left(H_z^*+\sin\alpha\right)^2\left(v^{*2}+u^{*2}\right) \tag{16}$$

$$\frac{\partial H^*_x}{\partial t^*}=\sin\alpha\frac{\partial u^*}{\partial x^*}+\frac{\partial}{\partial x^*}\left(u^*H^*_x\right)+\frac{1}{Rm}\left[\frac{\partial^2 H^*_x}{\partial x^{*2}}+\frac{\partial^2 H^*_x}{\partial z^{*2}}\right] \tag{17}$$

$$\frac{\partial H^*_z}{\partial t^*}=-\sin\alpha\frac{\partial u^*}{\partial z^*}+\frac{\partial}{\partial x^*}\left(u^*H^*_z\right)+\frac{1}{Rm}\left[\frac{\partial^2 H^*_z}{\partial x^{*2}}+\frac{\partial^2 H^*_z}{\partial z^{*2}}\right] \tag{18}$$

Where:

$$\left.\begin{aligned}&Er=\frac{\Omega\upsilon}{u_0^{\,2}},\quad \Pr=\frac{\mu C_p}{k},\quad Gr=\frac{g\beta\left(T_w-T_\infty\right)\theta\upsilon}{u_0^{\,3}},\quad m=\frac{\omega_e\tau_e}{H_0},\\ &M^2=\frac{\sigma\mu_e^{\,2}H_0^{\,2}\upsilon}{\rho u_0^{\,2}},\quad Ec=\frac{u_0^{\,3}}{C_p\Delta T},\quad R=\frac{\sigma B_0^{\,2}\mu}{\rho^2 C_p\Delta T},\end{aligned}\right\}$$

The initial and boundary conditions (9) in non-dimensional form are

$$t\le 0\quad u\left(x,z,t\right)=0,\quad v\left(x,z,t\right)=0,\quad T\left(x,z,t\right)=0 \tag{19}$$

$$t>0\quad \left.\begin{aligned}&u\left(x,z,t\right)=1, v\left(x,z,t\right)=0, T\left(x,z,t\right)=0\\ &u\left(\infty,z,t\right)=0, v\left(\infty,z,t\right)=0, T\left(\infty,z,t\right)=0\\ &H_x\left(x,z,t\right)=1, H_z\left(x,z,t\right)=0\end{aligned}\right\} \tag{20}$$

III. COMPUTATIONAL PROCEDURE

The set of nonlinear ordinary differential equations (13)-(18) with boundary conditions (19) and (20) are solved numerically using central difference finite method and Gauss Siedel iteration method. We make the centre values the subject i.e u_{ij}^k, v_{ij}^k

$$\begin{aligned}\left[\frac{2}{(\Delta z)^2}+\frac{2}{(\Delta x)^2}+\frac{M^2\left(H_z^*+\sin\alpha\right)^2}{1+m^2\left(H_z^*+\sin\alpha\right)^2}\right]u_{ij}^k&=\frac{1}{2\Delta z}\left[-\left(u_0u\right)_{ij+1}^k+\left(u_0u\right)_{ij-1}^k\right]+\frac{1}{(\Delta z)^2}\left[\left(u\right)_{ij+1}^k+\left(u\right)_{ij-1}^k\right]\\&+\frac{1}{(\Delta x)^2}\left[\left(u\right)_{i+1j}^k+\left(u\right)_{i-1j}^k\right]-\frac{1}{2\Delta x}\left[\left(u^2\right)_{i+1j}^k-\left(u^2\right)_{i-1j}^k\right]\\&+Gr\theta+2Erv_{ij}^k+M^2\left(H_z^*+\sin\alpha\right)^2\left[\frac{m\left(H_z^*+\sin\alpha\right)v_{ij}^k}{1+m^2\left(H_z^*+\sin\alpha\right)^2}\right]\end{aligned} \tag{21}$$

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$$\left[\frac{2}{(\Delta z)^2}+\frac{2}{(\Delta x)^2}+\frac{M^2\left(H_z^{\square}+\sin\alpha\right)^2}{1+m^2\left(H_z^{\square}+\sin\alpha\right)^2}\right]v_{ij}^k=\frac{1}{2\Delta z}\left[-\left(u_0v\right)_{ij+1}^k+\left(u_0v\right)_{ij-1}^k\right]+\frac{1}{(\Delta z)^2}\left[(u)_{ij+1}^k+(u)_{ij-1}^k\right]$$
$$+\frac{1}{(\Delta x)^2}\left[(v)_{i+1j}^k+(v)_{i-1j}^k\right]-\frac{1}{2\Delta x}\left[(uv)_{i+1j}^k-(uv)_{i-1j}^k\right]$$
$$-2Eru_{ij}^k-M^2\left(H_z^{\square}+\sin\alpha\right)^2\left[\frac{m\left(H_z^{\square}+\sin\alpha\right)u_{ij}^k}{1+m^2\left(H_z^{\square}+\sin\alpha\right)^2}\right] \tag{22}$$

The time advancement is done by invoking Gauss-Siedel iteration technique as below

$$u_{ij}^k=\frac{1}{A}\phi\left[u_{ij-1}^k,u_{ij-1}^k,u_{i-1j}^k,v_{ij}^k,v_{i+1j}^k,v_{i-1j}^k\right] \tag{23}$$

$$v_{ij}^k=\frac{1}{A}\psi\left[v_{ij-1}^k,v_{ij-1}^k,v_{i-1j}^k,u_{ij}^k,u_{i+1j}^k,u_{i-1j}^k\right] \tag{24}$$

For all i in [1,n-1] and j in [1,m-1] where

$$A=\left[\frac{2}{(\Delta z)^2}+\frac{2}{(\Delta x)^2}\pm\frac{M^2\left(H_z^{\square}+\sin\alpha\right)^2}{1+m^2\left(H_z^{\square}+\sin\alpha\right)^2}\right] \tag{25}$$

For the energy equation we make θ_{ij}^k the subject

$$\left[\frac{2}{\Pr(\Delta z)^2}+\frac{2}{\Pr(\Delta x)^2}\right]\theta_{ij}^k=-\frac{1}{2\Delta z}\left[\left(u_0\theta\right)_{ij+1}^k-\left(u_0\theta\right)_{ij-1}^k\right]-\frac{1}{2\Delta x}\left[(u\theta)_{i+1j}^k-(u\theta)_{i-1j}^k\right]$$
$$+\frac{Ec}{4(\Delta z)^2}\left[\left(u_{ij+1}^k+u_{ij-1}^k\right)^2+\left(v_{ij+1}^k+v_{ij-1}^k\right)^2\right]$$
$$+\frac{1}{\Pr(\Delta z)^2}\left[\theta_{ij+1}^k+\theta_{ij-1}^k\right]+\frac{1}{\Pr(\Delta x)^2}\left[\theta_{i+1j}^k+\theta_{i-1j}^k\right]$$
$$+R\left(H_z^{\square}+\sin\alpha\right)\left(v^{\square 2}+u^{\square 2}\right) \tag{26}$$

The time advancement is done by invoking Gauss-Siedel iteration technique as below

$$\theta_{ij}^{k+1}=\frac{1}{B}F\left[\theta_{ij+1}^k,\theta_{ij-1}^k,\theta_{i+1j}^k,u_{ij+1}^k,u_{ij+1}^k,v_{ij+1}^k,v_{ij-1}^k\right] \tag{27}$$

For all I in [1,n-1] and j in [1,m-1] where

$$B=\left[\frac{2}{\Pr(\Delta z)^2}+\frac{2}{\Pr(\Delta x)^2}\right] \tag{28}$$

The induction equation in spatial discretization in both x and z-direction

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$$\frac{(H_x)_{i,j}^{k+1}-(H_x)_{i,j}^{k}}{\Delta t}=\sin\alpha\left[\left(\frac{(u)_{i,j+1}^{k}-(u)_{i,j-1}^{k}}{2\Delta z}\right)\right]+\left[\frac{(uH_z)_{i,j+1}^{k}-(uH_z)_{i,j-1}^{k}}{2\Delta z}\right]$$
$$+\frac{1}{2Rm(\Delta x)^2}\left[(H_x)_{i+1,j}^{k}-2(H_x)_{i,j}^{k}+(H_x)_{i-1,j}^{k}\right]$$
$$+\frac{1}{2Rm(\Delta z)^2}\left[(H_x)_{i,j+1}^{k}-2(H_x)_{i,j}^{k}+(H_x)_{i,j-1}^{k}\right] \quad (29)$$

$$\frac{(H_z)_{i,j}^{k+1}-(H_z)_{i,j}^{k}}{\Delta t}=-\sin\alpha\left[\left(\frac{(u)_{i,j+1}^{k}-(u)_{i,j-1}^{k}}{2\Delta z}\right)\right]+\left[\frac{(uH_x)_{i,j+1}^{k}-(uH_x)_{i,j-1}^{k}}{2\Delta x}\right]$$
$$+\frac{1}{2Rm(\Delta x)^2}\left[(H_z)_{i+1,j}^{k}-2(H_z)_{i,j}^{k}+(H_z)_{i-1,j}^{k}\right]$$
$$+\frac{1}{2Rm(\Delta z)^2}\left[(H_z)_{i,j+1}^{k}-2(H_z)_{i,j}^{k}+(H_z)_{i,j-1}^{k}\right] \quad (30)$$

Making $(H_x)_{i,j}^{k}$ and $(H_z)_{i,j}^{k}$ the subject, we get

$$\frac{1}{Rm}\left[\frac{1}{(\Delta x)^2}+\frac{1}{(\Delta z)^2}\right](H_x)_{i,j}^{k}=-\sin\alpha\left[\frac{(u)_{i,j+1}^{k}-(u)_{i,j-1}^{k}}{2\Delta z}\right]+\left[\frac{(uH_x)_{i,j+1}^{k}-(uH_x)_{i,j-1}^{k}}{2\Delta z}\right]$$
$$+\frac{1}{2Rm(\Delta x)^2}\left[(H_x)_{i+1,j}^{k}+(H_x)_{i-1,j}^{k}\right] \quad (31)$$
$$+\frac{1}{2Rm(\Delta z)^2}\left[(H_x)_{i,j+1}^{k}+(H_x)_{i,j-1}^{k}\right]$$

$$\frac{1}{Rm}\left[\frac{1}{(\Delta x)^2}+\frac{1}{(\Delta z)^2}\right](H_z)_{i,j}^{k}=\sin\alpha\left[\frac{(u)_{i,j+1}^{k}-(u)_{i,j-1}^{k}}{2\Delta z}\right]+\left[\frac{(uH_z)_{i,j+1}^{k}-(uH_z)_{i,j-1}^{k}}{2\Delta z}\right]$$
$$+\frac{1}{2Rm(\Delta x)^2}\left[(H_z)_{i+1,j}^{k}+(H_z)_{i-1,j}^{k}\right] \quad (32)$$
$$+\frac{1}{2Rm(\Delta z)^2}\left[(H_z)_{i,j+1}^{k}+(H_z)_{i,j-1}^{k}\right]$$

The time advancement is done by invoking Gauss-Siedel iteration technique as below

$$(H_x)_{i,j}^{k+1}=\frac{1}{C}P\left[(H_x)_{i,j+1}^{k},(H_x)_{i,j-1}^{k},(H_x)_{i-1,j}^{k},(H_x)_{i+1,j}^{k},u_{i+1,j}^{k},u_{i,j}^{k},u_{i+1,j}^{k},u_{i-1,j}^{k}\right] \quad (33)$$

$$(H_z)_{i,j}^{k+1}=\frac{1}{C}P\left[(H_z)_{i,j+1}^{k},(H_z)_{i,j-1}^{k},(H_z)_{i-1,j}^{k},(H_z)_{i+1,j}^{k},u_{i+1,j}^{k},u_{i,j}^{k},u_{i+1,j}^{k},u_{i-1,j}^{k}\right] \quad (34)$$

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For all I in [1,n-1] and j in [1,m-1] and $C = \frac{1}{Rm}\left[\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta z)^2}\right]$

The space under consideration has been restricted to finite dimensions. Here a plate of height $x_{\max} = 50$ and $z_{\max} = 50$ have been considered.

A Matlab program was run for various values of velocity and temperature profiles for the finite difference equations 21,22, 23,24,26,27,31,32,33 and 34 subject to initial and boundary conditions 19 and 20.The process of computation is advanced until a steady state is approached with respect to the velocity fields by satisfying the convergence criterion

$$\sum_i \sum_j \left| A_{i,j}^{k+1} - A_{i,j}^{k} \right| < 10^{-5} \sum_i \sum_j \left| A_{i,j}^{k+1} \right| \tag{35}$$

Where $A_{i,j}^{k}$ stands for the velocity or temperature field.

In the numerical computation, special attention required to specify Δt in order to get to a steady state solution as soon as possible, yet small enough to avoid instabilities.
We set Δt

$$\Delta t = \lambda \times \min\left(\Delta x^2, \Delta z^2\right) \tag{36}$$

Where Δx and Δz are mesh sizes along the x and z directions, respectively. The stabilizer parameter λ is guessed by numerical experimentations in order to achieve convergence and stability of the solution procedure. A series of numerical experiments has shown that assigning the value 2 to λ is suitable for numerical computations.

Having solved for the velocity and temperature variables, one can compute the shear stress and heat transfer parameters at the vertical wall. The local skin friction components in the x and y directions denoted by τ_x and τ_y are defined as

$$\tau_x = \left.\frac{\partial u}{\partial z}\right|_{z=0}, \qquad \tau_y = \left.\frac{\partial v}{\partial z}\right|_{z=0} \tag{37}$$

The average values of the skin friction components with respect to the variable x are given by

$$\tau_{xav} = \frac{1}{x_{\max}} \int_0^{x_{\max}} \tau_x dx, \qquad \tau_{yav} = \frac{1}{x_{\max}} \int_0^{x_{\max}} \tau_y dx \tag{38}$$

The quantities τ_x and τ_y have been evaluated using a five-point finite difference formula for the first derivative, and then τ_{xav} and τ_{yav} have been computed using the Simpson's rule. In a similar way, the Nusselt number Nu and average Nusselt number Nu_{av} have been computed from the temperature field variable using the formulae

$$Nu = -\left.\frac{\partial \theta}{\partial z}\right|_{z=0}, \qquad Nu_{av} = \frac{1}{x_{\max}} \int_0^{x_{\max}} Nu dx \tag{39}$$

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IV. RESULTS AND DISCUSSIONS

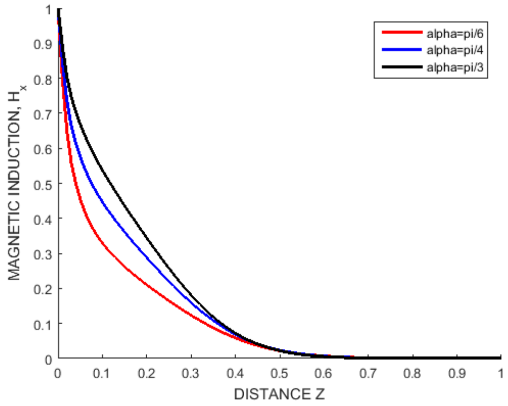

Fig 2: Variation of induced magnetic field $H_X$for various inclined angles with Pr=3.Rm=15, R=10

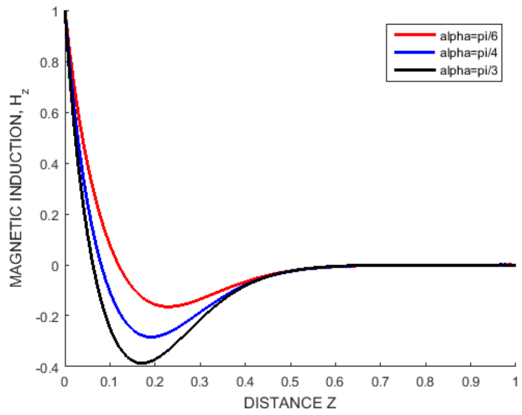

Fig 3: variation of induced magnetic field H_Z for various inclined angles with Pr=3, Rm=15, R=10

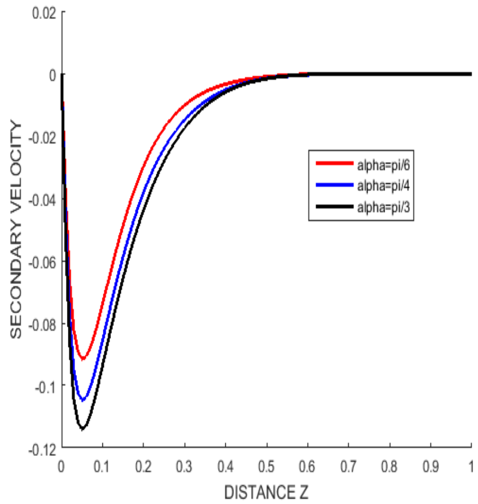

Fig 4: variation of secondary velocity for various values of alpha with Pr=3,Rm=15.R=10

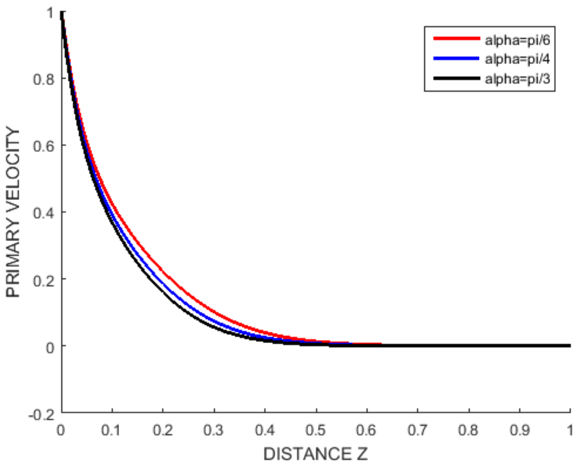

Fig 5: variation of primary velocity for various values of alpha with Pr=3,Rm=15, R=10

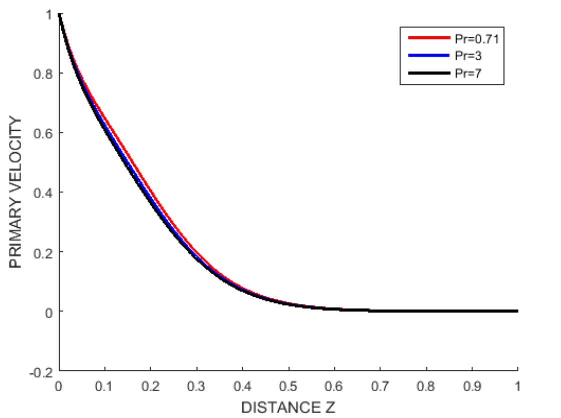

Fig 6: Variation of Primary velocity for various values of Prandtl Number (Pr) with $\alpha = \frac{\pi}{4}$,Rm=15,R=10

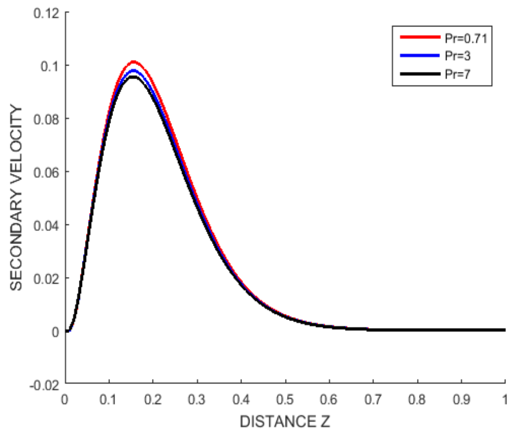

Fig 7: Variation of Secondary velocity for various values of Prandtl Number (Pr) with $\alpha = \frac{\pi}{4}$,Rm=15,R=10

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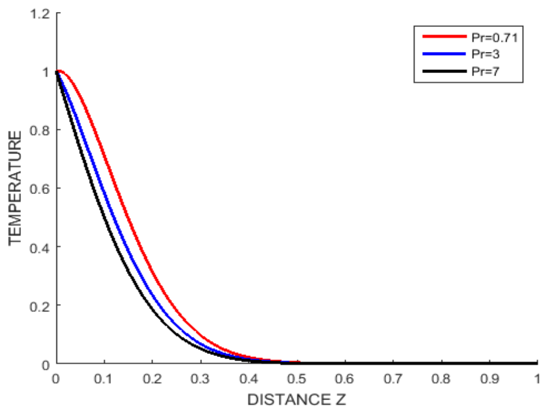

Fig 8: Variation of Temperature for various values of Prandtl Number (Pr) with $\alpha = \frac{\pi}{4}$,Rm=15.R=10

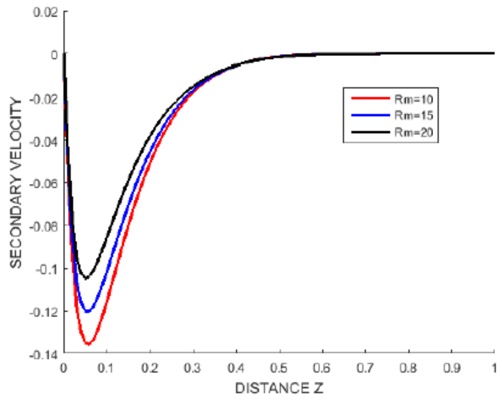

Fig 9: Variation of secondary velocity for various values of Magnetic. Reynolds Number (Rm) with $\alpha = \frac{\pi}{4}$, Pr=3,R=10

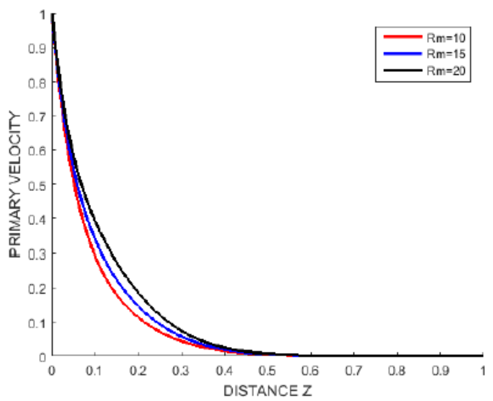

Fig 10: variation of primary velocity for various values of Magnetic. Reynold Number (Rm) with $\alpha = \frac{\pi}{4}$,Pr=3,R=10

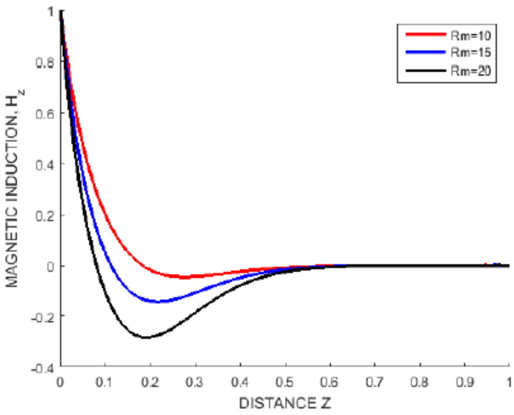

Fig 11: variation of Induced magnetic field (Hz) for various values of Magnetic Reynold Number (Rm) with $\alpha = \frac{\pi}{4}$,Pr=3,R=10

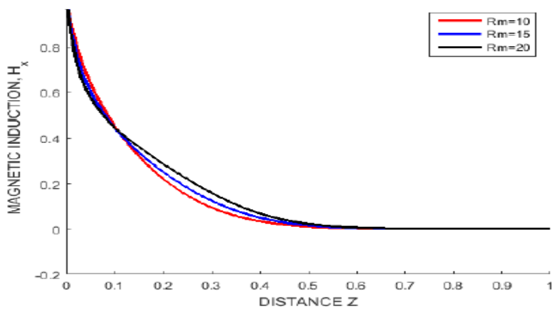

Fig 12: variation of Induced magnetic field(Hx) for various values of Magnetic. Reynold Number (Rm) with

$$\alpha = \frac{\pi}{4},\text{Pr=3,R=10}$$

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Finally the effects of angle of inclination, Prandtl Number, and Magnetic Reynolds number on the skin friction and Nusselt number are shown in the table below.

Table 1: Values of skin friction at the wall (τ) and heat flux (Nu)

α	Pr	Rm	τ_x	τ_y	Nu
$\frac{\pi}{6}$	3	15	0.1306	0.2834	-1.5564
$\frac{\pi}{4}$	3	15	0.2469	0.4424	-1.6514
$\frac{\pi}{3}$	3	15	0.3289	0.5643	-1.7229
$\frac{\pi}{4}$	0.71	15	0.1886	0.3977	-0.7508
$\frac{\pi}{4}$	7	15	0.4880	0.5461	-2.2915
$\frac{\pi}{4}$	3	10	0.2174	0.4424	-1.6514
$\frac{\pi}{4}$	3	20	0.2815	0.4499	-1.6331

The velocity and Temperature profiles for different parameters like angle of inclination of magnetic field α, Prandtl number (Pr) and Magnetic Reynolds Number(Rm) is shown in figures 2 to 12.Its evident from figure 2 that induced magnetic field along the direction of flow increases with increase in inclination angle α.The magnitude of induced magnetic field in the axial direction decreases as the angle increases as shown in figure 3.The effect of inclined magnetic field on an electrically conducting fluid give rise to a resistive type of force called Lorentz force. This force has tendency to slow down the motion of the fluid and to increase its temperature as shown in figures 5.The secondary velocity decreases as the inclination angle increases as shown in figure 4.In Figure 6 we observe that primary velocity decreases with increase in Prandtl number. Physically, this is true because the increase in the Prandtl number is due to increase in viscosity of the fluid which makes the fluid thick and hence causes a decrease in velocity of the fluid. The magnitude of the secondary velocity decreases with increase in Prandtl number as shown in figure 7.Temperature profiles decreases with increase in Prandtl number as shown in figure 8.An increase in Prandtl number causes a decrease in thermal diffusivity. Thermal diffusivity represent how fast heat diffuses through a material and is defined as ratio of heat conducted to heat stored i.e $\alpha = k/\rho c_p$ .Therefore a decrease in α causes a decrease in k. From Figure 10 we observe that the higher the Magnetic Reynolds number the higher the velocity profiles. At large Magnetic Reynolds numbers the inertia forces which are proportional to density and velocity of the fluid are large relative to magnetic diffusivity. The secondary velocity decreases with increases with increase in magnetic Reynolds number as shown in figure 9.Induced magnetic field along the z direction decreases as the magnetic Reynolds number increases as shown in figure 11.Figure 12 shows that induced magnetic field decreases with increase in magnetic Reynolds number within the boundary layer region but outside the boundary layer region the fluid velocity is small hence induced magnetic field increases.

From table 1, we note that an increase of angle of inclination increases skin friction. This is because the effect of increase in angle is to retard the flow since there is additional resistance called magnetic viscosity. An increase in Prandtl number causes an increase in both τ_x and τ_y .Increase in Prandtl number decreases both the primary velocity and secondary velocity and hence shear resistance increases. An increase in Magnetic Reynolds number causes a decrease in magnetic diffusivity and as a consequence increases the shear stress. Average Nusselt number decreases with increase in the angle of inclination. This implies that heat transfer is by conduction. The

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Nusselt number increases as the Prandtl number increases. Similar observation is made with respect to magnetic Reynolds number.

V. CONCLUSION

From the results above we have seen that the parameters in the governing equations affect the velocity and temperature profiles. Consequently their effects alter the skin friction and the rate of heat transfer. First, the induced magnetic field both in x and z direction is affected by angle of inclination; it increases along the direction of flow and decreases along the z direction. Also the as the angle increases skin friction τ_x, τ_y and local Nusselt number increases. Secondly, as the inclination angle increases both primary and secondary velocity decreases. Thirdly, as the Prandtl number increases, primary, secondary and Temperature profiles decreases. On the other hand increase in Magnetic Reynolds number increases both primary and secondary velocity. The Magnetic Reynolds number reduces induced magnetic field along z direction, but along x direction it reduces within the boundary layer and reverses outside the boundary layer. The Authors recommend that the same problem can be studied incorporating mass transfer with variable surface temperature.

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NOMENCLATURE

u, v, w: Velocity components in x, y, z directions respectively

Ec :Eckert number

Pr: Prandtl number

C_P : Specific heat at constant pressure

R: Joule heating parameter

k: Thermal conductivity

Nu: Nusselt number

T: Temperature

J: Joule heating parameter

M: Magnetic number

m: Hall parameter

Er: Rotational Parameter

ν: Kinematical viscosity, m^2 / s

θ: Dimensionless temperature, K

μ : Dynamic Viscosity

ω : Cycloton Frequency/s

μ_e :Magnetic permeability,H/m

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Ω : Angular velocity m/s

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