MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2018/2019 ACADEMIC YEAR
SECOND YEAR SECOND SEMESTER

SCHOOL OF SCIENCE AND INFORMATION SCIENCES
BACHELOR OF SCIENCE (APPLIED STATISTICS & COMPUTING)

COURSE CODE: STA 429
COURSE TITLE: APPLIED MULTIVARIATE ANALYSIS

DATE: 29/04/2019 **TIME: 11:00 AM – 1:00 PM**

INSTRUCTIONS TO CANDIDATES

1. Answer Question ONE and any other Two questions.
2. Show all the workings clearly
3. Do not write on the question paper
4. All Examination Rules Apply.

This paper consists of 4 printed pages. Please turn over.

Question One (30 Marks)

a) Define the following terms

i) Multivariate analysis (**2 Marks)**

ii) The rank of a matrix (**2 Marks)**

b) The joint probability distribution function of random variables Y_1 *and* Y_2 is given in the table below;

Y_1 \ Y_2	0	1
-1	0.24	0.06
0	0.16	0.10
1	0.40	0.00

Find $\underline{\mu}$ and Σ for the random vector $\underline{Y} = (Y_1, Y_2)$ **(7 Marks)**

c) Determine the eigenvalues and eigenvectors of the matrix A given as;

$$A = \begin{pmatrix} [illegible] & 2 \\ [illegible] & -2 \end{pmatrix}$$

(6 Marks)

d) Let A be 3×3 symmetric matrix of constants and $\underline{X}'$ be a 3-dimensional vector. Given that the quadratic form Q is of the form;

$$Q = \underline{x}'A\underline{x} = 3x_1^2 + 13x_2^2 + x_3^2 + 10x_1x_2 + 2x_1x_3$$

Identify the matrix A **(4 Marks)**

e) Suppose that the variance-covariance matrix is given as;

$$\Sigma = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 9 & -3 \\ 2 & -3 & 25 \end{bmatrix}$$

Find the $Diag(\Sigma)$ and hence or otherwise find ρ . **(4 Marks)**

f) A sample of 101 observations is drawn from a bivariate normal distribution with unknown $\underline{\mu}$ and Σ . The results are; such that $s = \begin{bmatrix} 210.54 & 126.99 \\ 126.99 & 119.68 \end{bmatrix}$ and $\bar{x} = \begin{bmatrix} 55.24 \\ 34.97 \end{bmatrix}$

Test $H_0 : \mu' = (60, 50)$ at $\alpha = 1\%$ level of significance. **(5 Marks)**

Question Two (20 Marks)

Consider the bivariate normal distribution given by;

$$f(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{X_1-\mu_1}{\sigma_1}\right)^2 - 2\rho\left(\frac{X_1-\mu_1}{\sigma_1}\right)\left(\frac{X_2-\mu_2}{\sigma_2}\right) + \left(\frac{X_2-\mu_2}{\sigma_2}\right)^2\right]}$$

i) Obtain the marginal density function of X_1 **(10 Marks)**

ii) Obtain the conditional density function of X_1 given $X_2 = x_2$

(10 Marks)

Question Three (20 Marks)

a) Let $\underline{Y}' = (Y_1, Y_2, Y_3)$ be a random vector with a trivariate normal distribution, $N_3(\underline{\mu}, \Sigma)$ with $\underline{\mu}' = (2,1,2)$ and $\Sigma = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 0 \\ 1 & 0 & 1 \end{pmatrix}$. Find

i) The distribution of $x = 3y_1 + 2y_2 - y_3$ **(3 Marks)**

ii) The joint distribution of $x = (x_1, x_2)'$ where $x_1 = y_1 + y_2 + y_3$ and $x_2 = y_1 - y_2$

(5 Marks)

b) The random vector $\underline{X}' = (x_1, x_2, x_3, \ldots, x_n)$ has a joint probability density function given by;

$$f(x) = \begin{cases} \dfrac{n!}{\prod_{i=1}^{k} x_i!} \prod_{i=1}^{k} p^{x_i}, & where\ 0 \le p_i \le 1,\ \sum_{i=1}^{k} p_i = 1\ and\ \sum_{i=1}^{k} x_i = n \\ 0, & elsewhere \end{cases}$$

i) Obtain the moment generating function of $\underline{X}$

(5 Marks)

ii) Use the moment generating function of $\underline{X}$ to find the mean vector and the variance-covariance matrix of $\underline{X}$.

(7 Marks)

Question Four (20 Marks)

The municipal waste water treatment plants are required by law to monitor the discharges into rivers on regular basis.

Measurements on Biochemical oxygen Demand (BOD) and suspended solids (SS) are obtained from 10 sample splits from the two labs and the results posted as follows,

Commercial Lab		**State Lab**	
BOD	**SS**	**BOD**	**SS**
6	27	25	15
6	23	28	13
18	64	36	22
8	44	35	29
11	30	15	31
34	75	44	64
28	26	42	30
71	124	54	64
43	54	34	56
33	30	29	20

i) Obtain the matrix of difference Δ (**5 Marks)**

ii) Obtain the mean vector and S_d (**5 Marks)**

iii) Determine if the two labs are the same with respect to the measurements of the discharges, i.e Test $H_0 : \underline{\mu}_1 = \underline{\mu}_2$ against $H_1 : \underline{\mu}_1 \neq \underline{\mu}_2$ at $\alpha = 0.05$ level of significance. (**10 Marks)**

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