MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATION
2018/2019 ACADEMIC YEAR
SECOND YEAR SECOND SEMESTER EXAMINATIONS

FOR

THE DEGREE OF BACHELOR SCIENCE (MATHEMATICS), APPLIED STATISTICS WITH COMPUTING AND EDUCATION (SCIENCE, ARTS AND SPECIAL NEEDS)

COURSE CODE: MAT 2212
COURSE TITLE: REAL ANALYSIS I

DATE 18TH APRIL 2019 TIME: 1100 – 1300HRS

INSTRUCTIONS TO CANDIDATES

1. This paper contains **FOUR** (4) questions

2. Answer question **ONE (1)** and any other **TWO** (**2**) questions
3. Do not forget to write your Registration Number.

QUESTION 1 (30MARKS)

a) Define power set $P(X)$ of a set X and hence show that the power set $P(\mathbb{N})$ of $\mathbb{N}$ is uncountable
5marks

b) Given that $A=\left\{\frac{1}{n}:n\in\mathbb{N}\right\}$. Determine $\sup A$, $\inf A$ and state whether the maximum and minimum of A exists.
4marks

c) Show that if $x\neq 0$, then $x^2>0$ and hence deduce that $1>0$
4marks

d) Prove that for a subset A of $\mathbb{R}$ that is bounded below $\inf A$ is unique
4**marks**

e) Prove that $\sqrt{2}$ is irrational.
5marks

f) Using the ratio test determine whether the following series converge or diverge $\sum_{n=1}^{\infty}\frac{n^2}{2^n}$
3 marks

g) Define the function $\rho:\mathbb{R}^2\times\mathbb{R}^2\to\mathbb{R}$ by $\rho(x,y)=|x_1-y_1|+|x_2-y_2|$ where $x=(x_1,x_2)$, $y=(y_1,y_2)$. Show that ρ is a metric on $\mathbb{R}^2$
5marks

QUESTION 2 (20MKS)

a) Let A and B be non-void subsets of $\mathbb{R}$ that are bounded above. Show that $\sup(A+B)=\sup(A)+\sup(B)$
5marks

b) Show that the empty set ϕ is a subset of any other set
3marks

c) Show that every convergent sequence is Cauchy
5marks

d) Define a continuous function and hence determine whether the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \begin{cases} \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$
3marks

e) Show that every Cauchy sequence is bounded
4marks

QUESTION 3 (20MKS)

f) Show that a point $p \in X$ is a limit point of $E \subseteq X$ iff there exists a sequence $(x_n)_1^{\infty}$ of distinct points of E with $x_n \neq p \ (\forall\, n \in \mathbb{N})$ such that $\lim_{n \to \infty} x_n = p$
10marks

g) Show that if the sequences (x_n) and (y_n) are convergent and $x_n \leq y_n$ for all $n \in \mathbb{N}$, then $\lim_{x \to \infty} x_n \leq \lim_{x \to \infty} y_n$
5marks

h) If $f(x) = \begin{cases} \frac{1}{x} & x \neq 0 \\ \text{[illegible]} & x = 0 \end{cases}$ find $f'(x)$.
5marks

QUESTION 4 (20MKS)

a) Test for convergence in the following series

i. $\sum_{n=1}^{\infty} 2^{-n}$ ii. $\sum_{n=1}^{\infty} (-1)^{n+1}$ iii. $\sum_{n=1}^{\infty} n^{-1}$
9marks

b) Classify the monotonic sequences below.

i. $x_n = n^3$

ii. $x_n = (-1)^{n+1}$

iii. $x_n = \dfrac{1}{n}$

iv. $x_n = 2$ $\forall\, n$��

4marks

c) Binary operation $*$ on the set of all real numbers **R** is defined by $x * y = |x - y|$. Show that $*$ is commutative but not associative

2marks

d) Define the terms

i. A metric space

1mark

ii. Neighbourhood

1mark

iii. A convergent sequence

1mark

iv. Monotonic sequences

1mark

v. Uniformly continuous function

1mark

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