MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2016/2017 ACADEMIC YEAR
YEAR III SEMESTER I

SCHOOL OF MATHEMATICAL AND PHYSICAL SCIENCES
BACHELOR OF SCIENCE

COURSE CODE:STA 2217
COURSE TITLE: MATHEMATICAL STATISTICS II

DATE:3RD DEC,2018 **TIME: 08:30-10:30 A.M**

INSTRUCTIONS TO CANDIDATES

1. Answer Question **ONE** and any other TWO questions.
2. All Examination Rules Apply.

Question 1(30 Marks)

a) (i) Define the term order statistics. (2mks)

(ii) Let X_1, X_2 be a random sample from a distribution with density function.

$f(x) = e^{-x}, 0 < x < \infty$

What is the density of Y= min { X_1, X_2 }. (3mks)

(iii) Consider 2 independent and identically distributed random variables X and Y whose pdfs are;

$f(x) = 6x(1-x), 0 < x < 1$ and

$f(y) = 3y^2, 0 < y < 1$ respectively.

Find the pdf of $Z = XY$. (5mks)

b) The bivariate probability distribution of the random variables X and Y is summarized in the following table.

Y

X		0	1	2	3
	0	k	6k	9k	4k
	1	8k	18k	12k	2k
	2	k	6k	9k	4k

(i) Find k. (3mks)

(ii) Obtain the marginal distributions of X and Y. (4mks)

(iii) Find the conditional distribution of X given Y =2. (3mks)

(iv) State with a reason whether or not X and Y are independent. (2mks)

c) The daily number of road traffic accidents, Y, in a certain town can be modelled by a Poisson distribution which has probability mass function.

$$P(Y = k) = \frac{e^{-\lambda}\lambda^k}{k!}, k = 0,1,2,...; \lambda > 0$$

(i) Show that the probability generating function (pgf) of Y is $e^{-\lambda(1-t)}$. (3mks)

(ii) Use the pgf to show that E(Y) = Var (Y) = λ. (5mks)

Question 2 (20 Marks)

(a) The joint probability density function of the random variables X and Y.

$$f(x,y) = \frac{1}{2\pi}\exp\{-\frac{1}{4}(x-1)^2 - (y - \frac{1}{4}(1+x))^2\}, -\infty < x, y < \infty$$

(i) Use integration to show that X has the normal distribution with mean 1 and variance 2.

(7mks)

(ii) Use integration to show that the moment generating function of X is $M_X(t) = \exp\{t+t^2\}$

(7mks)

(iii) Use the moment generating function to find $E(X^3)$. (6mks)

Question 3 (20 Marks)

a) Define the terms probability generating function (pgf) and the moment generating function (mgf) of a random variable X and give the relationship between these two functions.

(3mks)

b) The random variable X has the binomial distribution with parameters $n(n > 3)$ and $p(0 < p < 1)$.

(i) Show that the probability generating function of X is; (4mks)

$$\pi t = (pt + 1 - p)^n, -\infty < t < \infty$$

(ii) Use (i) to show that $E(X) = np$ and $Var(X) = np(1-p)$. (5mks)

(iii) Find $E(X^3)$. (3mks)

(iv) Now suppose that $X_1, X_2, ..., X_m$ are independent random variables and X_i has the binomial distribution with parameters n and p for $i = 1, 2, ..., m$. Let $Y = \sum_{i=1}^{m} X_i$ .Find the pgf of Y, and hence deduce the distribution of Y. (5mks)

Question 4 (20 Marks)

Suppose that the discrete random variables X and Y independently follow Poisson distributions such that;

$$P(X = x) = \frac{e^{-\theta}\theta^{x}}{x!}, x = 0,1,...$$

and $$P(Y = y) = \frac{e^{-\theta}\theta^{y}}{y!}, y = 0,1,...$$

Show that the random variables $X + Y$ also follows a Poisson distribution. (8mks)

(i) Now suppose that $X + Y = Z$, where Z is some non-negative integer. Determine $P(X = x / X + Y = z)$ for all possible values of x. What is the conditional distribution of X, given that $X + Y = Z$? (7mks)

(ii) When preparing student handouts, lecturers A and B make typing errors at random, A at a rate of 1.5 errors per page and B at a rate of 0.5 errors per page. A course handout consists of 6 pages typed by lecturer A and 12 pages typed by lecturer B. It is found to contain a total of 14 typing errors. Show that the probability that lecturer A made at least 10 of the mistakes on this handout is 0.279. (5mks)