MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

YEAR 3: SEMESTER 1

SCHOOL OF SCIENCE AND INFORMATION SCIENCES

BACHELOR OF SCIENCE/BACHELOR OF EDUCATION-SCIENCE/ARTS/SPECIAL EDUCATION

COURSE CODE: MATH 3117

COURSE TITLE: ADVANCED CALCULUS

DATE: 13/12/2018 TIME: 8.30-10.30 A.M (2 HOURS)

INSTRUCTIONS TO CANDIDATES

1. Answer Question **ONE** and any other **TWO** questions

2. Do not write anything on this question paper

QUESTION ONE (30 MARKS)

(a) Evaluate

(i) $\int_{-\infty}^{\infty} x^3 e^{-x^4}\, dx.$ (4 mks)

(ii) $\int_0^3 \frac{1}{\sqrt{3-x}}\, dx$ (4 mks)

(b) Use the direct comparison test to determine whether the integral $\int_1^{\infty} \frac{x}{\sqrt{1+x^6}}\, dx$ converges or diverges. (3 mks)

(c) (i) Apply Taylor's theorem to the series e^x to estimate the value of the integral $\int_0^1 e^{x^2}\, dx$ (6 mks)

(ii) Expand $y = \log x$ in powers of $x - 1$, hence evaluate the value of $\log 1.1$ correct to 4 decimal places. (5 mks)

(d) Evaluate

(i) the surface integral $\iint_s x^2yz\, ds$ where s is the part of the plane $z = 1 + 2x + 3y$ that lies above $(0,0)$ to $(2,3)$. (5 mks)

(ii) $\int_c f(x,y)ds$ if $f(x,y) = xy, c\colon x = \cos t, y = \sin t, 0 \le t \le \frac{\pi}{2}$. (3 mks)

QUESTION TWO (20 MKS)

(a) Evaluate $\int_{(0,1)}^{(1,2)} (x^2 - y)dx + (y^2 + x)dy$ along

(i) a straight line from $(0,1)$ to $(1,2)$ (4 mks)

(ii) straight lines from $(0,1)$ to $(1,1)$ and then from $(1,1)$ to $(1,2)$ (8 mks)

(iii) the parabola $x = t$, $y = t^2 + 1$ (3 mks)

(b If c is the segment from $(0,0,0)$ to $(1,2,3)$, find $\int_c xe^{yz}\, ds$. (5 mks)

QUESTION THREE (20 MKS)

(a) Verify green's theorem in the plane for $\oint_c \ (2xy - x^2)dx + (x + y)dy$

where c is the closed curve of the region bounded by $y = x^2$ and $y^2 = x$.(14 mks)

(b) (i)Show that the area bounded by a simple closed curve c is given by

$\frac{1}{2}\oint_c \ xdy - ydx.$ (3 mks)

(ii) Find the area of the ellipse $x = a\ cos\ \theta$, $y = b\ sin\ \theta$ for $0 \leq \theta \leq 2\pi$. (3 mks)

QUESTION FOUR (20 MKS)

(a) Evaluate$\oint_c \ y^2 dx + x^2\, dy$ where c is the triangle bounded by lines

$x = 0; x + y = 1; y = 0.$ (3 mks)

(b) Find$\iint_s \ curl\ F.ds$, where s is the hemisphere $x^2 + y^2 + z^2 = 9$,

$F(x, y, z) = 2y \cos z\, i + e^x sin\ zj + xe^y k$, $z \geq 0$ in upward orientation. (6 mks)

(c) Using Gauss divergence theorem , evaluate $\iint_s \ F.N\ ds$, where

$F = 4xi - 2y^2 j + z^2 k$ and s is the surface bounded by the region

$x^2 + y^2 = 4$, $z = 0$, $z = 3$. (11 mks)