MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS

2023/ 2024 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES.

MASTERS (APPLIED STATISTICS)

COURSE CODE: STA 8214

COURSE TITLE: NON-PARAMETRIC METHODS

DATE: **TIME:**

INSTRUCTIONS TO CANDIDATES

Answer Question ONE and any other TWO questions

This paper consists of FOUR printed pages. Please turn over.

QUESTION ONE (20 MARKS)

The table below show students' performance during the first week of class. Use $\alpha = 0.05$ for your desired level of confidence.

90	72	90
64	95	89
74	88	100
77	57	35
100	64	95
65	80	84
90	100	96

(a) You are required to compute:

i. The mean and standard deviation (4 mks)

ii. The Kurtosis. (4 mks)

iii. The standard error of Kurtosis. (3 mks)

(b) Use the Kurtosis and the standard error of the Kurtosis to find the Z-score. (2 mks)

(c)

i. Use the mean and standard deviation to find the skewness. (3 mks)

ii. Find the standard error of skewness. (2 mks)

iii. Use the skewness and the standard error of the skewness to find the z-score. (2 mks)

QUESTION TWO (20 MARKS)

(a)

i. Let $X_1, X_2, \ldots, X_n$ be iid continuous random variables with pdf f and cdf F. Show that the density of the maximum is given by:

$f_n(x) = nf(x)F(x)^{n-1}$ (6 mks)

ii. Prove also that the density of the minimum is given by:

$f_1(x) = nf(x)\big(1 - F(x)\big)^{n-1}$ (6 mks)

(b)

i. Given $X_1, X_2, \ldots, X_n$ to be iid variables and X_k as the smallest k^{th} of X and $X_{(n)}$ is the largest X. Define:

a. $X_{(1)}$ the smallest X. (1 mk)

b. $X_{(n)}$ the largest X. (1 mk)

ii. For $X_1, X_2, \ldots, X_n$ iid random variables with pdf f and cdf F, show that the density of the k^{th} order statistic is given by:

$$f_k(x) = nf(x)\binom{n-1}{k-1}F(x)^{k-1}\left(1 - F(x)\right)^{n-k}$$ (6 mks)

QUESTION THREE (20 MARKS)

(a) Let X be a Beta distribution given as $X \sim Beta(r, s)$, you are required to show that:

i. $E(X) = \frac{r}{r+s}$ (5 mks)

ii. $Var(X) = \frac{rs}{(r+s+1)(r+s)^2}$ (7 mks)

(b)

i. Let $X_1, X_2, \ldots$, Xn be **iid** $Exp(\lambda)$ where by the minimum i.e.

$$f_1(x) = \min(X_1, X_2, \ldots, X_n) \sim Exp(\lambda_1 + \lambda_2 + \ldots + \lambda_n) = Exp(n\lambda)$$

Show that this minimum is given by:

$f_1(x) = n\lambda e^{-n\lambda x}$ (5 mks)

ii. Give the density of $X_{(n)}$, the maximum of the exponentials. (3 mks)

QUESTION FOUR (20 MARKS)

(a) Given that $Y = \frac{X_1+X_2+\cdots+X_n}{N}$, we consider that Y is an approximation to the mean of X and by observing values $x_1, x_2, \ldots x_n$ for $X_1, X_2, \ldots, X_n$ then observed sample mean is $y = \frac{x_1+x_2+\cdots+x_N}{N}$. Show that:

i. $\mu_Y = \mu_X$ (3 mks)

ii. $\sigma_Y^2 = \frac{\sigma_X^2}{N}$ (4 mks)

(b) Bb

i. A city health department wishes to determine if the mean bacteria count per unit volume of water at a lake beach is within the safety level of 200. A researcher collected 10 water samples of unit volume and found the bacteria count to be:

175	190	215	198	184
207	210	193	196	180

Assuming that the measurements are a sample from a normal population, by showing all steps of your work including the hypotheses, test yourself if data indicate that bacteria count is within the safety level. Take $\alpha = 0.01$ (8 mks)

ii. A brochure inviting subscriptions for a new diet program states that the participants are expected to lose over 22 pounds in five weeks. Suppose that, from the data of the five-week weight losses of 56 participants, the sample mean and standard deviation are found to be 23.5 and 10.2 pounds, respectively. Could the statement in the brochure be substantiated on the basis of these findings? Test with $\alpha = 0.05$. (5 mks)