

MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATION

2023/2024 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

SCHOOL OF SCIENCE AND INFORMATION SCIENCES

MASTER OF SCIENCE (APPLIED STATISTICS)

COURSE CODE: STA 8212

COURSE TITLE: STOCHASTIC PROCESSES II

DATE: 20/5/2024

TIME 11:00 – 1300 HOURS

INSTRUCTIONS TO CANDIDATES.

- i. Question one is compulsory
- ii. Answer any other two questions.

QUESTION 1 (20MARKS)

- a) Consider the case $\lambda = n\lambda$, $\lambda_n = n\lambda$ and $\lambda = m$. Find the difference – differential equation (5mks)
- b) If we have n state out of a possible number of a states that we have $\lambda = (a-n)\lambda$ and $\lambda_n = n\lambda$, $0 \leq n \leq a$ which is birth – death process which models a power-supply problem. Find its difference – differential equation.
- c) Suppose X and Y are integral valued random variables with joint probability distribution $P_n(x = j, y = k) = P_{jk}$, $J = 0, 1, 2, \dots$ and $k = 0, 1, 2, \dots$ and $\sum_j \sum_k P_{jk} = 1$
- Find the joint pgf of x and y (2mks)
 - Pgf of marginal distribution (3mks)
 - Pgf of conditional distribution (2mks)
- d) State three methods of finding the mean and variance (3mks)

QUESTION 2 (20MARKS)

Let x have a negative binomial distribution given by,

$$P_r(x = k) = p_k = \binom{k-r}{r-1} p^r (1-p)^{k-r}, k = r, r+1, \dots$$

Find

Pgf of x

Mean of x

Variance of x

QUESTION THREE (20 MARKS)

- a) Let x have a pdf $p_r(x = k)$ $k = 0, 1, 2, 3, \dots$
With pgf $P(s) = \sum p_k s^k$ and $q_k = \text{pr}(x = k) = p_{k+1} + p_{k+2}$
With generating function $\emptyset(s) = \sum q_k s^k$ ($\sum q_k \neq 1$)
Express $\emptyset(s)$ in terms of $p(s)$ (10mks)
- b) Let $x_i, i = 1, 2, 3, \dots$ be iid random variable with $\text{pr}(x_i = p_k)$ and pgf $p(s) = \sum p_k s^k$ for $i = 1, 2, 3, \dots$. Then pgf $H(s)$ of S_N is given by the compound function $G(p(s))$ i.e. $H(s) = \sum \text{pr}(S_N = j) s^j = G(p(s))$ proof (10mks)

QUESTION FOUR (20 MARKS)

Consider a series of Bernoulli trials with probability of success p . suppose that x denotes the number of failures preceding the first success and y the number of failures following the first success and preceding the second success obtain the joint pdf of x and y , mean of x and y and variance of x and y .