MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS

2023/2024 ACADEMIC YEAR

SECOND SEMISTER EXAM

SCHOOL OF PURE APPLIED AND HEALTH SCIENCES

MASTER OF SCIENCE IN APPLIED STATISTICS

COURSE CODE: STA 8211

COURSE TITLE: MULTIVARIATE ANALYSIS II

DATE: 04/5/24 TIME:0830-1030HRS

INSTRUCTIONS TO CANDIDATES

Answer Any Three Questions

Question One (20 Marks)

a) Define canonical correlation **(2Marks)**

b) What is the significance of canonical correlation **(3Marks)**

c) Consider the following variance co variance matrix;

	x_1	x_2	y_1	y_2
x_1	100	0	0	0
x_2	0	1	0.75	0
y_1	0	0.75	0	100

Obtain the correlation co-efficient between the first pair of canonical variables **(5marks)**

d) The variance covariance matrix between 5 yield attributing parameters x_1, x_2 , x_3 , x_4, x_5 and four quality attributes y_1, y_2 , y_3 , y_4 based on a sample size 200 is given as; Perform canonical correlation analysis and interpret your results. **(10marks)**

	x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3	y_4
x_1	1.000	0.754	0.690	-0.44	0.702	-0.605	-0.48	0.78	-0.152
x_2	0.754	1.00	-0.710	-0.515	0.412	-0.722	-0.419	0.542	-0.100
x_3	-0.690	-0.710	1.000	0.323	-0.444	0.737	0.361	-0.546	0.172
x_4	-0.440	-0.515	0.323	1.000	-0.334	0.527	0.461	-0.393	-0.019
x_5	0.702	0.412	-0.444	-0.334	1.00	-0.383	-0.505	0.737	-0.148
y_1	-0.605	-0.722	0.737	0.527	-0.383	1.000	0.251	-0.490	0.250
y_2	-0.480	-0.419	0.361	0.461	-0.505	0.251	1.000	-0.434	-0.079
y_3	0.780	0.542	-0.546	-0.393	0.737	-0.490	-0.434	1.000	-0.163
y_4	-0.152	-0.100	0.172	-0.019	-0.148	0.250	-0.079	-0.163	1.000

Question Two: (20 Marks)

a) Define the following terms as used in principal component analysis giving examples;

i) Quadratic form **(3marks)**

ii) Bilinear form **(3marks)**

b) Let A= $\begin{bmatrix} 3 & 6 & -1 \\ 6 & 9 & 4 \\ -1 & 4 & 3 \end{bmatrix}$

i) Find the spectral decomposition of A **(8Marks)**

ii) Establish the relationship between; the eigenvalues, the determinants of the matrices, and the trace of the matrices. **(6Marks)**

Question Three (20 Marks)

a. Define discriminant analysis **(2 marks)**
b. What is the aim of discriminant analysis **(2 marks)**
c. Describe one example of discriminant analysis **(3marks)**
d. Consider two normal populations; $p_1: N(\mu_1, \sigma_1^2)$, and $p_2: N(\mu_2, \sigma_2^2)$, suppose that $\mu_1 = 0, \mu_2 = 1, \sigma_1 = 1$ and $\sigma_2 = 0.5$. Use discriminant analysis to allocate the variable of x to either the first population or the second population. **(5Marks)**
e. Compute the linear discriminant projection for the following two-dimensional data set.
$\pi_1 = (x_1, x_2) = (4,2), (2,4), (2,3), (3,6), (4,4)$
$\pi_2 = (x_1, x_2) = (9,10), (6,8), (9,5), (8,7), (10,8)$ **(8marks)**

Question four (20 marks)

a) consider the matrix R= $\begin{bmatrix} 1 & 0.975 & 0.613 \\ 0.975 & 1 & 0.620 \\ 0.613 & 0.620 & 1 \end{bmatrix}$ where $p = 3$ and $k = 1$,

i) Find the degrees of freedom d **(2marks)**
ii) What is the implication when $d > 0, d < 0,$ and $d = 0$ **(2marks)**
iii) Evaluate q_1^2 and $\hat{\varphi}_{11}$ **(2marks)**

b) Consider the table below and answer the following questions,

	Estimated factor Loadings		Communalities	Specific variance
Variable	$\hat{q}_1$	$\hat{q}_2$	h_j^2	$\hat{\varphi}_{ij} = 1 - h_j^2$
Taste	0.56	0.82		
Good buy for money	0.78	-0.53		
Flavor	0.65	0.75		
Suitable for snack	0.94	-0.11		
Provides a lot of energy	0.80	-0.54		

i) Obtain the values of h_j^2 and $\hat{\varphi}_{ij}$ **(6 marks)**

ii) Use the results above to perform factor analysis and give your conclusions **(4marks)**

iii) Use the estimated factor loadings and the specific variance to ascertain that two factor model provides a good fit for the data if the original data is given by the matrix,

1.00 0.02 0.96 0.42 0.01
0.02 1.00 0.13 0.71 0.85
0.96 0.13 1.00 0.50 0.11
0.42 0.71 0.50 1.00 0.79
0.01 0.85 0.11 0.79 1.00 **(4marks)**