MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2023/2024 ACADEMIC YEAR
FOURTH YEAR SECOND SEMESTER

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES
BACHELOR OF SCIENCE APPLIED STATISTICS WITH COMPUTING

COURSE CODE: STA 4246-1
COURSE TITLE: MULTIVARIATE ANALYSIS

DATE: 18 /4/2024 **TIME: 0830-1030HRS**

INSTRUCTIONS TO CANDIDATES

1. Answer **Question ONE** and any other **Two** questions.
2. Show all the workings clearly
3. Do not write on the question paper
4. All Examination Rules Apply.

Question One (20 Marks)

a) The moment generating function of a random vector $\underline{X}' = [X_1, X_2]$ is given by

$$M(t_1, t_2) = \exp\left[2t_1 + 3t_2 + 2t_1^2 - 2t_1t_2 + 8t_2^2\right]$$

Determine the: (i). Mean vector (ii). Correlation matrix **(6marks)**

b) Let A be a 3x3 symmetric matrix of constants and be a dimensional vector. Given that the quadratic form Q is of the form

$$Q(\underline{x}) = \underline{X}'A\underline{X} = 3x_1^2 + 13x_2^2 + x_3^2 + 10x_1x_2 + 2x_1x_3$$

(i). Identify A

(ii). Show that A is positive definite matrix. **(4 marks)**

c). Given that:

$$P = \begin{bmatrix} 1 & \frac{1}{6} & \frac{1}{5} \\ \frac{1}{6} & 1 & -\frac{1}{5} \\ \frac{1}{5} & -\frac{1}{5} & 1 \end{bmatrix} \quad and \quad V = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 25 \end{bmatrix}$$

Determine Σ **(4 marks)**

d) Let X_1, X_2 have a bivariate normal distribution with parameters.

$\mu_1 = 5, \mu_2 = 10, \sigma_1^2 = 1, \sigma_2^2 = 25 \quad and \quad \rho > 0$. If $P(4 < X_2 < 16 / X = 5) = 0.954$,

determine ρ. **(6 marks)**

Question Two (15 Marks)

Let $\underline{X}' = [X_1, X_2, X_3]$ be a trivariate normal random vector with mean vector

$\underline{\mu} = [1, 2, 3]$ and variance- covariance matrix $\Sigma = \begin{bmatrix} 5 & 2 & 3 \\ 2 & 3 & 0 \\ 3 & 0 & 2 \end{bmatrix}$. Find the following.

(i). Mean and variance of $Y = X_1 - 2X_2 + X_3$.

(ii). Mean vector $\underline{\mu}_Y$ and variance-covariance matrix Σ_Y of $\underline{Y}' = [Y_1, Y_2]$ where

$$Y_1 = X_1 + X_2 + X_3$$
$$Y_2 = X_1 + X_2$$

(iii). Distribution of Y_1 *and* Y_2.

(iv). Are Y_1 *and* Y_2 independent? **(15 marks)**

Question Three (15 Marks)

A random sample of size 10 is taken from a bivariate normal population with mean $\underset{\sim}{\mu}$ and variance- covariance Σ both unknown. The sample mean vector $\underset{\sim}{\bar{x}}$ and the inverse of the sample variance- covariance matrix S^{-1} were obtained as

$$\underset{\sim}{\mu} = \begin{bmatrix} 3.32 \\ 1.74 \end{bmatrix} \quad S^{-1} = \begin{bmatrix} 0.266 & -0.099 \\ -0.099 & 0.189 \end{bmatrix}$$

At $\alpha = 5\%$, test the hypothesis

$$H_0 : \underset{\sim}{\mu} = \begin{bmatrix} 5 \\ 3 \end{bmatrix} \quad Vs \quad H_1 : \underset{\sim}{\mu} \neq \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$ **(8 marks)**

b) Let **X** have the covariance matrix.

$$\Sigma = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}$$

Determine P *and* $V^{\frac{1}{2}}$ **(7 marks)**

Question Four (15 Marks)

a). Consider the data matrix

$$X = \begin{bmatrix} -1 & 3 & -2 \\ 2 & 4 & 2 \\ 5 & 2 & 3 \end{bmatrix}$$

Calculate the generalized sample variance and the total sample variance. **(8 marks)**

b) . Given the data matrix.

$$X = \begin{bmatrix} 3 & 4 & 5 & 4 \\ 6 & 4 & 7 & 7 \end{bmatrix}$$

Determine the (i). Covariance matrix

(ii). Generalized variance

(iii). Total variance **(7 marks)**

Question Five (15 Marks)

a) Let $\underline{X}' = [X_1, X_2]$ be a random vector whose components are random variables with joint probability density function

$$f(x_1, x_2) = \begin{cases} k(2 - x_1 - x_2), & 0 \le x_1 \le 2, \quad 0 \le x_2 \le 1 \\ 0 & , elsewhwere \end{cases}$$

where k is a scalar. Determine the

(i). value of k

(ii). Marginal density of X_1 *and* X_2.

(iii). Correlation matrix of $\underline{X}$ **(8 marks)**

b). Using the matrix

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -2 \\ 2 & 2 \end{bmatrix}$$

Calculate $A'A$ and obtain its eigenvalues. Is . $A'A$ positive definite?. **(7 marks)**

// **END//**