MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY

EXAMINATION

2023/2024 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SCHOOL OF SCIENCE AND INFORMATION SCIENCES

BACHALOR OF SCIENCE IN APPLIED STATISTICS

COURSE CODE: STA 4243-1

COURSE TITLE: MEASURE AND PROBABILITY THEORY

DATE: **TIME:**

INSTRUCTIONS TO CANDIDATES

i. Question **ONE** is compulsory
ii. Answer any other **TWO** question

QUESTION ONE 20 MARKS

State three properties of a measure. (3mks)

a) State three properties of the outer measure of a set E (3mks)
b) The Lebesque outer measure of an empty set is zero. Proof (3mks)
c) The outer measure of an interval equals its length
d) If $M^*(A)$=0, Prove that $M^*(AUB) = M^*(B)$ For any set B (3mks)
e) Let $\{Ei: 1 \leq i \leq n\}$ be a finite collection of disjoint measurable sets. If A ≤ R, then $M^*(\bigcup_{i=1}^{\infty}(AUE_i) = M^*(An\{\bigcup_{i=1}^{\infty} E_i\}) = \sum_{i=1}^{n} M^*\,(AnE_i)$
   Proof. (5mks)

QUESTION TWO 15 MARKS

1. Every interval is Lebesque measurable. Proof (15mks)

QUESTION THREE 15 MARKS

2. If (E_i) is a sequence of measurable sets, then
   $M(\bigcup_{i=1}^{\infty} E_i) = \sum_{i=1}^{n} M(E_i)$ Proof (5mks)

b) If E_i is a sequence of measurable sets then $\bigcup_{i=1}^{\infty} E_i$ and $\prod_{i=1}^{\infty} E_i$ are measurable sets. Proof (10mks)

QUESTION FOUR 15 MARKS

3. For any sequence of sets E_n, $M^*(\bigcup_{n=1}^{\infty} E_n) \leq \sum_{i=1}^{\infty} M^*(\,E_n)$ Proof (10mks)
   a) The set (0,1) is countable, Proof. (3mks)
   b) Every interval is not countable, Proof. (2mks)