MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS

2023/2024 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES

BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: MAT 3228-1

COURSE TITLE: FOURIER ANALYSIS

DATE: APRIL, 2024 **TIME: 2 HOURS**

INSTRUCTIONS TO CANDIDATES

1. Answer **Question One (Compulsory)** and **ANY** other **Two Questions.**
2. All Examination Rules Apply.

This paper consists of 4 printed pages. Please turn over

Question One (20 Marks)

a. If $f(x)$ is even function show that

i. $b_n = 0$ **(3 Marks)**

ii.

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx$$

(**4 Marks**)

b. Obtain a Fourier series for the periodic function $f(x)$ defined

$$f(x) = \begin{cases} -k; & -\pi < x < 0 \\ +k; & 0 < x < \pi \end{cases}$$

The function is periodic outside of this range with period 2π **(4 Marks)**

c. Find the Fourier series of the function

$$f(x) = \begin{cases} 0 \;\; if -2 < x < -1 \\ k \; if \quad -1 < x < 1 \\ 0 \; if \quad\quad 1 < x < 2 \end{cases} \quad P = 2L = 4, L = 2$$

(4 Marks)

d. Using complex form find the Fourier series of the function

$$f(x) = \frac{a\sin x}{1 - 2a\cos x + a^2} \; ; \;\; |a| < 1$$

(5 Marks)

Question Two (15 Marks)

a. Prove that

i.

$$\frac{1}{2} + \cos t + \cos 2t + \ldots\ldots\ldots. + \cos Mt = \frac{\sin\left(M + \frac{1}{2}\right)t}{2\sin\frac{1}{2}t}$$

(5 Marks)

ii.

$$\frac{1}{\pi}\int_0^{\pi}\frac{\sin\left(M + \frac{1}{2}\right)t}{2\sin\frac{1}{2}t}dt = \frac{1}{2} \; ; \; \frac{1}{\pi}\int_{-\pi}^{0}\frac{\sin\left(M + \frac{1}{2}\right)t}{2\sin\frac{1}{2}t}dt = \frac{1}{2}$$

(4 Marks)

b. Show that for all positive integers M,

$$\frac{a_0^2}{2} + \sum_{n=1}^{M} (a_n^2 + b_n^2) \leq \frac{1}{L} \int_{-L}^{L} \{f(x)\}^2 dx$$

where a_n and b_n are the Fourier coefficients corresponding to $f(x)$ and $f(x)$ is assumed piecewise continuous in $(-L, L)$ **(6 Marks)**

Question Three (15 Marks)

a. If the series

$$A + \sum_{n=1}^{\infty} \left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L} \right)$$

converges uniformly to $f(x)$ in $(-L, L)$. Show that for $n = 1,2,3, \ldots \ldots \ldots$

i.

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos\frac{n\pi x}{L} dx$$

(**3 Marks**)

ii.

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin\frac{n\pi x}{L} dx$$

(**2 Marks**)

iii. $A = \frac{a_0}{2}$ **(1 Marks)**

b. A sinusoidal voltage, $E \sin \omega t$ where t is time is passed through a half wave rectifier that clips the negative portion of the wave. Find the Fourier series of the resulting periodic function

$$u(t) = \begin{cases} 0 & if -L < t < 0 \\ E \sin \omega t & if \quad 0 < t < L \end{cases} \quad P = 2L = \frac{2\pi}{\omega}\ ; L = \frac{\pi}{\omega}$$

(4 Marks)

c. Find the two half-range expansions of the triangle with function given below using even and odd periodic extensions

$$f(x) = \begin{cases} \frac{2k}{L} x & if\ 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L - x) & if\ \frac{L}{2} < x < L \end{cases}$$

(5 Marks)

Question Four (15 Marks)

a. Using complex form, find the Fourier series of the function

$$f(x) = \sin x = \begin{cases} -1 & ; \ -\pi \le x \le 0 \\ 1 & ; \quad 0 < x \le \pi \end{cases}$$ **(5 Marks)**

b. Prove

i.

$$\int_{-L}^{L} \cos\frac{m\pi x}{L}\cos\frac{n\pi x}{L}\,dx = \int_{-L}^{L} \sin\frac{m\pi x}{L}\sin\frac{n\pi x}{L}\,dx = \begin{cases} 0 & if \ \ m \neq n \\ L & if \ \ m = n \end{cases}$$

(5 Marks)

ii.

$$\int_{-L}^{L} \sin\frac{m\pi x}{L}\cos\frac{n\pi x}{L}\,dx = 0 \qquad\qquad if \ \ m \neq n \ or \ m = n$$

(5 Marks)

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