MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS

2023/2024 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES

BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: MAT 3225-1

COURSE TITLE: COMPLEX ANALYSIS II

DATE: APRIL, 2024 **TIME: 2 HOURS**

INSTRUCTIONS TO CANDIDATES

1. Answer **Question One (Compulsory)** and **ANY** other **Two Questions.**
2. All Examination Rules Apply.

This paper consists of 4 printed pages. Please turn over

Question One (20 Marks)

a. Evaluate

$$\int_c \frac{e^z}{(z-4)(z-6)}\,dz$$

When

i. c is the circle $|z| = 3$ **(1 Mark)**

ii. c is the circle $|z| = 5$ **(2 Marks)**

iii. c is the circle $|z| = 7$ **(3 Marks)**

b. Distinguish between an isogonal and conformal mapping **(2 Marks)**

c. Find the poles of the following function

$f(z) = \frac{6z^2-15}{z^6+64}$ **(3 Marks)**

d. Prove the Schwarz- Christoffel transformation

$w = A\int (z-x_1)^{\frac{\alpha_1}{\pi}-1}(z-x_2)^{\frac{\alpha_2}{\pi}-1} \ldots\ldots\ldots\ldots\ldots..(z-x_n)^{\frac{\alpha_n}{\pi}-1}dz + B$

Which maps the interior R of the polygon on the $w-plane$ on to the upper half R' of the $z-plane$ and the boundary of the of the polygon on to the real axis of the $z-plane$ **(4 Marks)**

e. Evaluate

$$\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$$

(5 Marks)

Question Two (15 Marks)

a. Given the function $f(z) = \frac{1}{z^4+16}$

i. Find the poles of the function $f(z)$ **(4 Marks)**

ii. Calculate the residues of $f(z)$ at its poles **(4 Marks)**

iii. Hence evaluate

$$\int_{-\infty}^{\infty} \frac{dx}{x^4+16}$$

(3 Marks)

b. Evaluate

$$\int_0^{2\pi} \frac{d\theta}{13 + 5\cos\theta}$$

(4 Marks)

Question Three (15 Marks)

a. What is the sum of all the exterior angles of a closed polygon of n sides? **(2 Marks)**

b. Show that for closed polygons, the sum of the exponents

$$\left(\frac{\alpha_1}{\pi} - 1\right) + \left(\frac{\alpha_2}{\pi} - 1\right) + \cdots \ldots \ldots \ldots \ldots \ldots \ldots \ldots .. + \left(\frac{\alpha_n}{\pi} - 1\right) = -2$$

in Schwarz- Christoffel transformation **(3 Marks)**

c. Show that the power series $z - \frac{1}{2}z^2 + \frac{1}{3}z^3 - \cdots \ldots \ldots ..$ may analytically continue to a wider region by means of the series

$$\ln 2 - \frac{1-z}{2} - \frac{(1-z)^2}{2.\,2^2} - \frac{(1-z)^3}{2.\,2^3} - \cdots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$

(4 Marks)

d. Prove that the series

$$\sum_0^{\infty} \frac{z^n}{2^{n+1}} \; and \; \sum_0^{\infty} \frac{(z-i)^n}{(2-i)^{n+1}}$$

are convergent and they are analytic continuation of each other **(6 Marks)**

Question Four (15 Marks)

a. Determine the poles and the residues at the poles of the function

i. $f(z) = \frac{3z^3+2}{(z-2)(z^2+16)}$ **(2 Marks)**

ii. $f(z) = \frac{2z+3}{z^2+25}$ **(2 Marks)**

iii. Hence evaluate

$$\int_{c} \frac{2z+3}{z^2+25}\,dz$$

where c is the circle $|z| = 6$ **(5 Marks)**

b. Evaluate

$$\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^3}$$

(6 Marks)

THIS IS THE LAST PRINTED PAGE