MAASAI MARA UNIVERSITY REGULAR UNIVERSITY EXAMINATIONS
2023/2024 ACADEMIC YEAR
SCHOOL OF PURE, APLLIED AND HEALTH SCIENCES
SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS,PHYSICS

MAT:2216-1 ADVANCED CALCULUS
Instructions to candidates:
Answer Question 1 and TWO other Questions .
All Symbols have their usual meaning

DATE: TIME:

Question 1(20 Marks)

(a) Let f be a function of two or more variables. Explain what is meant by a partial derivative of f and give an example to illustrate (**2 Marks**)

(b) Determine the partial derivatives of the following functions

(i) $f(x, y) = x^3y + x^2y^4 - \frac{2}{xy^3}$ (**2 Marks**)

(ii) $g(u, v) = (e^{2u+v} - v^3)^{\frac{1}{2}}$ (**2 Marks**)

(c) If $w = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$, determine $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial w}{\partial z^2}$ (**3 Marks**)

(d) State what is meant by total differential of a function. Illustrate with the function

$h(x, y) = x^3y + xy^2 + y^3$ (**3 Marks**)

(e) Prove that the series

$$1 + \frac{1}{2} + \frac{1}{3} + \ldots$$

is divergent (**3 Marks**)

Hence prove that

$\frac{1}{1000} + \frac{1}{2000+1000} + \frac{1}{3000+1000} + \ldots$ is divergent (**2 Marks**)

(i) Prove that $\sum_{n=1}^{\infty} \frac{1}{1000n + 1}$ is divergent (**3 Marks**)

Question 2 (15 Marks)

(a) A sequence a_n is defined by

$$a_n = \begin{cases} 1 + (\frac{1}{4})^n, & \text{if } n \equiv 1 (\text{mod } 3), \\ 2 + (\frac{1}{2})^{2n}, & \text{if } n \equiv 2 (\text{mod } 3), \\ 3 - (\frac{1}{3})^n, & \text{if } n \equiv 0 (\text{mod } 3) \end{cases}$$

(i) Write down the first 7 terms of the sequence (**2 Marks**)

(ii) Determine all the accumulation points of the sequence (**2 Marks**)

(b) State and prove the Cauchy Criterion for the convergence of sequences (**6 Marks**)

(c) Show that the following sequences are convergent and find their limits

(i) $a_n = \frac{100{,}000n}{1+n^2}$ (**2 Marks**)

(ii) $b_n = \frac{3^n+(-2)^n}{3^{n+1}+(-2)^{n+1}}$ (**3 Marks**)

Question 3 (15Marks)

(a) (i) Prove that if the series $\sum a_n$ converges then its nth term tends to zero (**4 Marks**)

(ii) Show by an example that the converse in (i) is false (**2 Marks**)

(b) Determine the sum of the geometric series

(i) $\sum_{n=1}^{k} r^n$ (**3 Marks**)

(ii) $\sum_{n=1}^{\infty} (\frac{1}{z})^n$ (**2 Marks**)

(c) Using suitable comparison show that $\sum_{k=1}^{\infty} \frac{1}{k!}$ is a convergent series and state an upper bound on its value (**4 Marks**)

Question 4 (15 Marks)

(a) Evaluate $\int\int r^2 \cos\theta dr d\theta$ in the region $1 \leq r \leq 3$ and $\frac{\pi}{3} \leq \theta \leq \frac{\pi}{2}$ (**4 Marks**)

(b) Define the term "improper" integral and give example to illustrate (**4 Marks**)

(c) Show that $\int_0^b e^{-x^2} dx$, $b > 0$, exists and is finite (**4 Marks**)

(d) Evaluate $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$ (**3 Marks**)