MAASAI MARA UNIVERSITY

MAIN EXAMINATION 2024/2025 ACADEMIC YEAR
SECOND YEAR FIRST SEMESTER EXAMINATIONS

FOR

THE DEGREE OF BACHELOR OF SCIENCE MATHEMATICS

MAT 2215: INTRODUCTION TO ALGEBRAIC STRUCTURES

DATE: APRIL, 2024 DURATION: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This paper contains **FIVE** (5) questions
2. Answer question **ONE (1)** and any other **TWO** (**2**) questions
3. Do not forget to write your Registration Number.

QUESTION ONE (20 MARKS)

a) Distinguish between an onto mapping and a 1-1 mapping (2 marks)

b) Let G = {1, 2, 3, 4, 5, 6} under multiplication modulo 7.

  (i) Form the multiplication table for G (2 marks)

  (ii) State the inverses 5 and 6 (1 marks)

c) The operation ∘and * are defined on such that

$$x \circ y = 3x + 4y$$

$$x * y = 2y - x$$

Determine;

(i) If the operations are associative (2 marks)

(ii) If the identity element under the operations exist for each operation (3 marks)

(iii) Show that ∘is left distributive over * (2 marks)

d) Define a set (1 mark)

e) Discuss the following terms as used in algebraic structures;

  i. Subgroup (1 mark)

  ii. Abelian group (1 mark)

f) Express explicitly all the elements of the set;

  i. A = {X: 1 ≤ x ≤ 12, x is odd} (1 mark)

  ii. B = { y : $y^3 - 4y = 0$} (2 marks)

g) Define the order of a group (1 marks)

h) Define a cyclic group. (1 marks)

QUESTION TWO (15 MARKS)

a) Define a group (2 marks)

b) Let G be the set of all real numbers except -1 i.e. $G = \{R\backslash\{-1\}\}$. Define * on G by

$$a * b = a + b - ab.$$

(i) Show that G is a subgroup under * (5 marks)

(ii) Is G an abelian group? (5 marks)

(iii) Solve the equation 2*3*x=7 (3 marks)

QUESTION THREE 1 (15 MARKS)

a) Define an inverse mapping (2 marks)

b) Define a ring with its five addition axioms (5 marks)

c) Show that $\mathbb{Z}_3$ is a field while $\mathbb{Z}_4$ is not (4 marks)

d) Show that the operation * on $\mathbb{Z}$ define by $a * b = a + b + 3 \quad \forall\, a, b \in \mathbb{Z}$ satisfies the closure and associativity properties (4 marks)

QUESTION FOUR ((15 MARKS)

a) Let $f: R \to R$ be define be $f(x) = \frac{x}{x+1}$

(i) Find $f(-5)$ and the domain of f (2 marks)

(ii) Find f^{-1} and $f^{-1}(1)$ (4 marks)

(iii) Show that f is 1-1 (3 marks)

b) Let $f: \mathbb{Z} \to \mathbb{Z}$, $g: \mathbb{Z} \to \mathbb{Z}$ be defined by $f(x) = x2 + 2x + 1$and $g(x) = 4x - 5$. Show that $gof \neq fog$ (4 marks)

c) The tables below are Cayley tables of the ring (R, +, •)

+	a	b	c	d	e	f	g
a	e	a	d	g	f	c	b
b	a	b	c	d	e	f	g
c	d	c	a	e	g	b	f
d	g	d	e	f	b	a	c
e	f	e	g	b	c	d	a
f	c	f	b	a	d	g	e
g	b	g	f	c	a	e	d

•	a	b	c	d	e	f	g
a	f	b	d	a	g	e	c
b	b	b	b	b	b	b	b
c	d	b	c	c	f	a	e
d	a	b	c	d	e	f	g
e	g	b	e	e	d	c	a
f	e	b	f	f	c	g	d
g	c	b	g	g	a	d	f

Determine the additive and multiplications identities (2 marks)

QUESTION FIVE (15 MARKS)

a) State Lagrange's theorem on subgroups of a finite group and use the subgroups of

i. Z_{12}

ii. S_3

to illustrate (4 marks)

(i) Let G = {1, -1, I, -i} and H = {1, -1}. Is (H, •) a subgroup of (G, •)? (3 marks)

(ii) What is a left coset of H in G? (2 marks)

(iii) Find all the disjoint cosets of H in G from b(ii) above (2 marks)

(iv) Give the definition of a field and hence state the examples of field with an infinite number of elements. (4 marks)