MAASAI MARA UNIVERSITY
REGULAR UNIVERSITY EXAMINATIONS
2023/2024 ACADEMIC YEAR
FIRST YEAR SECOND SEMESTER
SCHOOL OF PURE APPLIED AND HEALTH SCIENCES
THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND EDUCATION
MAT 1206-1: DIFFERENTIAL CALCULUS

Instructions to candidates:

Answer Question 1. And any other TWO.

All Symbols have their usual meaning

DATE: TIME:

Question 1(20 Marks)

(a) Investigate the behavior of $f(\theta) = sin\theta$ as $\theta \to 0$. **(2 Marks)**

(b) Use δ and ϵ definition of a limit to show that

$$\lim_{x \to 4} \frac{x^2 - 2x - 8}{x - 4} = 6.$$

(5 Marks)

(c) Determine where the function

$$f(x) = \begin{cases} \frac{1}{x+2}, & \text{if } x \neq -2 \text{ ;} \\ 1, & \text{if x } = -2, \end{cases}$$

is discontinuous. **(3 Marks)**

(d) Find $\frac{dy}{dx}$ by implicit differentiation given that

$$x^2y + ay^2 = b,$$

where a and b are fixed constants. **(4 Marks)**

(e) Find the linearization of the function $f(x) = \sqrt{x+3}$ at $a = 1$ **(3 Marks)**

(f) Use parametric differentiation to determine $\frac{dy}{dx}$ given that $x = r(\theta - sin\theta), y = r(1 - cos\theta)$ **(3 Marks)**

Question 2 (10 Marks)

(i) Find correct to 6 dp the root of the equation $cosx = x$ using Newton Raphson method. Take the initial approximation to be $x_1 = 1$ **(7 Marks)**

(ii) If $1200cm^3$ of material is available to make a box with a square base and an open top, find the largest possible volume of the box **(8 Marks)**

Question 3 (15 Marks)

(a) At what point on the curve

$$y = \sqrt{1 + 2x}$$

is the tangent line perpendicular to the line $6x + 2y = 1$? **(5 Marks)**

(b) Under certain circumstances a rumor spreads according to the equation

$$p(t) = \frac{1}{1 + ae^{-kt}},$$

where $p(t)$ is the proportion of the population that has heard the rumor at time t and a, k are positive constants. Determine **(4 Marks)**

i. $\lim_{x\to\infty} p(t)$,

ii. the rate of spread of the rumor when $t = 0$ and $k = a = 1$.

(c) A ladder 12 meters long leans against a wall. The foot of the ladder is pulled away from the wall at the rate $\frac{1}{2}$ m/min. At what rate is the top of the ladder falling when the foot of the ladder is 4 meters from the wall? **(3 Marks)**

(d) Find the limit

$$\lim_{x\to 0} \frac{sin5x}{3x}.$$

(3 Marks)

Question 4 (15 Marks)

(a) The motion of a spring that is subject to a fractional force or a damping force is often modeled by the product of an exponential function and a sine or cosine function. Suppose that the equation of motion of a point on such a spring is

$$s(t) = 2e^{-1.5t} sin2\pi t,$$

where s is measured in centimeters and t is measured in seconds. Find the velocity after t seconds. **(4 Marks)**

(b) Given that $y = x^4 + 2x^3 - 3x^2 - 4x + 4$, find

i. the intervals on which y is increasing and decreasing,

ii. the maximum and minimum values of y. **(9 Marks)**

(c) Prove that $f(x) = \frac{\ln x}{x}$ has a horizontal asymptote $y = 0$. **(2 Marks)**