MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2023/2024 ACADEMIC YEAR
SECOND YEAR FIRST SEMESTER

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES
MASTER OF SCIENCE IN APPLIED STATISTICS & COMPUTING

COURSE CODE: STA 8215
COURSE TITLE: TIME SERIES ANALYSIS

DATE: 30/1/2024 **TIME: 0830-1130 HRS**

INSTRUCTIONS TO CANDIDATES

INSTRUCTIONS

i) **Answer any FOUR Questions**

ii) **Show all the workings clearly**

iii) **Do not write on the question paper**

QUESTION ONE (15 MARKS)

a) Suppose that annual sales (in millions of dollars) of the Acme Corporation follow the AR (2) model with $X_t = 5 + 1.1X_{t-1} - 0.5X_{t-2} + e_t$ and , $\sigma_e^2 = 2$

i) If sales for 2005, 2006, and 2007 were \$9 million, \$11 million, and \$10 million, respectively, forecast sales for 2008 and 2009.

ii) Show that $\Psi_1 = 1.1$ for this model.

iii) Calculate 95% prediction limits for your forecast in part (a) for 2008.

iv) If sales in 2008 turn out to be \$12 million, update your forecast for 2009. **(7 Marks)**

b) Find the ACF and PACF and plot the ACF, ρ_k for k=0,1,2,3,4,5 for each of the following models given that $e_t \sim WN(0, \sigma_e^2)$:

i) $X_t - 0.5X_{t-1} = e_t$

ii) $X_t + 0.98X_{t-1} = e_t$

iii) $X_t - 1.3X_{t-1} + 0.4X_{t-2} = e_t$

iv) $X_t - 1.2X_{t-1} + 0.8X_{t-2} = e_t$

Verify whether each of the models is stationary and/or invertible. **(8 Marks)**

QUESTION TWO (15 MARKS)

a) Consider the following model: $(1 - B)^2 X_t = (1 - 0.3B - 0.5B^2)e_t$.

i) Is the model for X_t stationary? why?

ii) Let $W_t = (1 - B)^2 X_t$ Is the model for W_t stationary? Why?

iii) Find the ACF for the second order difference W_t. **(5 Marks)**

b) For each of the following modes:

i) $(1 - \phi_1 B)(X_t - \mu) = e_t$

ii) $(1 - \phi_1 B - \phi_2 B^2)(X_t - \mu) = e_t$

iii) $(1 - \phi_1 B)(1 - B)X_t = (1 - \theta_1 B)e_t$

I) Find the l-step ahead forecast $\widehat{X_t}(l)$ of X_{t+l}.

II) Find the variance of the l-step ahead forecast error for $l = 1,2$ and 3. **(10 Marks)**

QUESTION THREE (15 MARKS)

a) Consider the IMA (1,1) model $(1 - B)X_t = (1 - \theta \mathrm{B})e_t$.

i) Write down the forecast equation that generates the forecasts

ii) Find the 95% forecast limits produced by this model.

iii) Express the forecasts as a weighted average of the previous observations. **(6 Marks)**

b) Assume that 100 observations from an AR (2) models $X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + e_t$ gave the following sample ACF: $\widehat{\rho_1} = 0.8$, $\widehat{\rho_2} = 0.5$, and $\widehat{\rho_1} = 0.4$. Estimate ϕ_1 and ϕ_2. **(5 Marks)**

c) For the model $(1 - 0.2B)(1 - B)X_t = (1 - 0.5\mathrm{B})e_t$,evaluate the first four ψ weights of the model when expressed as an $MA(\infty)$ model. Is the behavior of the ψ weights of the model what you would expect, given the type of model?
(4 Marks)

QUESTION FOUR (15 MARKS)

a) A sales series was fitted by the ARIMA (2,1,0) model $(1 - 1.4B - 0.7B^2)(1 - B)X_t = e_t$ where , $\sigma_e^2 = 58000$ and the last 5 observations are 560,580, 640, 770 and 800.

i) Calculate the forecasts of the next 3 observations.

ii) Find the 95%forecast limits for the forecasts in (a).

iii) Find the eventual forecast function. **(8 Marks)**

b) Assume that 100 observations from an ARMA (1,1) model, $X_t - \phi_1 X_{t-1} = e_t - \theta_1 e_{t-1}$ gave the following estimates: $\widehat{\sigma}_x^2$=10, $\widehat{\rho_1} = 0.523$, and $\widehat{\rho_2} = 0.418$. Find initial estimates for ϕ_1, θ_1 and , σ_e^2.

(7 Marks)

QUESTION FIVE (15 MARKS)

a) Consider an AR (2) model $(1-\phi_1 B-\phi_2 B^2)(X_t-\mu)=e_t$, where $\phi_1=1.2, \phi_2=-.6, \mu=65,$ and $, \sigma_e^2=1$. Suppose we have the observations $X_{76}=60.4, X_{77}=58.9, X_{78}=64.7$, $X_{79}=70.4$, and $X_{80}=62.6$.

i) Forecast X_{81}, X_{82}, X_{83}, and X_{84}.

ii) Find the 95%forecast limits for the forecasts in (a).

iii) Suppose that the observations at t=81 turns out to be $X_{81}=62.2$. Find the updated forecasts for X_{82}, X_{83}, and X_{84}. **(8 Marks)**

b) Consider the following processes:

i) $(1-B)^2 X_t=e_t-0.81e_{t-1}+0.3e_{t-2}$

ii) $(1-B)X_t=(1-0.5\text{B})e_t$

Express each of the above processes in the AR representation by actually finding and plotting the π weights.

(7 Marks)

QUESTION SIX (15 MARKS)

a) Consider the model $(1-0.43B)(1-B)X_t=e_t$ and the observations $X_{49}=33.4$, and $X_{50}=33.9$.

i) Compute the forecast $X_t(l)$, for $l=1,2,\ldots,10$, and their 90%forecast limits.

ii) What is the eventual forecast function for the forecasts made at t=50?

iii) At time t=51, X_{51}became known and equaled 34.1. Update the forecasts obtained in (a). **(9 Marks)**

b) The following data was obtained for a process X_t

t	1	2	3	4	5	6
X_t	29.33	19.98	29.00	31.03	32.68	30.12

Find the sample autocorrelation functions: r (1), r (2), r (3) and r (4). **(6 Marks)**

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