MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2023/2024 ACADEMIC YEAR
FIRST YEAR SECOND SEMESTER

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES
MASTER OF SCIENCE IN APPLIED STATISTICS & COMPUTING

COURSE CODE: STA 8213
COURSE TITLE: STOCHASTIC PROCESSES II

DATE: 29/1/2024 **TIME: 0830-1030 HRS**

INSTRUCTIONS TO CANDIDATES

i) Answer any FOUR Questions

ii) Show all the workings clearly

iii) Do not write on the question paper

QUESTION ONE (15 MARKS)

a) Machines in a factory break down at an exponential rate of six per hour. There is a single repairman who fixes machines at an exponential rate of eight per hour. The cost incurred in lost production when machines are out of service is $10 per hour per machine. What is the average cost rate incurred due to failed machines? **(4 Marks)**

b) Consider a machine that works for an exponential amount of time having mean $1/\lambda$ before breaking down; and suppose that it takes an exponential amount of time having mean $1/\mu$ to repair the machine. If the machine is in working condition at time 0, then what is the probability that it will be working at time $t = 10$? **(4 Marks)**

c) Beverly has a radio that works on a single battery. As soon as the battery in use fails, Beverly immediately replaces it with a new battery. If the lifetime of a battery (in hours) is distributed uniformly over the interval (30, 60),

i. then at what rate does Beverly have to change batteries?

ii. Suppose that Beverly does not keep any surplus batteries on hand, and so each time a failure occurs she must go and buy a new battery. If the amount of time it takes for her to get a new battery is uniformly distributed over (0,1), then what is the average rate that Beverly changes batteries?

(7 Marks)

QUESTION TWO (15 MARKS)

a) Suppose that customers arrive at a Poisson rate of one per every 12 minutes, and that the service time is exponential at a rate of one service per 8 minutes. What are the average number of customers in the system, L and the average amount of time a customer spends in the system, W? **(4 Marks)**

b) Consider a job shop that consists of M machines and one serviceman. Suppose that the amount of time each machine runs before breaking down is exponentially distributed with mean $1/\lambda$, and suppose that the amount of time that it takes for the serviceman to fix a machine is exponentially distributed with mean $1/\mu$. We shall attempt to answer these questions:

i. What is the average number of machines not in use?

ii. What proportion of time is each machine in use? **(6 Marks)**

c) Derive a renewal equation assuming that the interarrival distribution is uniform on interval (0, 1). Also present a solution for this renewal equation in the case when $t \leq 1$. **(5 Marks)**

QUESTION THREE (15 MARKS)

a) For an M/M/1 queue in steady state, what is the probability that the next arrival finds n in the system?

(3 Marks)

b) Potential customers arrive at a single-server station in accordance with a Poisson process with rate λ. However, if the arrival finds n customers already in the station, then he will enter the system with probability αn. Assuming an exponential service rate μ, set this up as a birth and death process and determine the birth and death rates. **(4 Marks)**

c) Suppose that potential customers arrive at a single-server bank in accordance with a Poisson process having rate λ. However, suppose that the potential customer will enter the bank only if the server is free when he arrives. That is, if there is already a customer in the bank, then our arriver, rather than entering the bank, will go home. If we assume that the amount of time spent in the bank by an entering customer is a random variable having distribution G, then,

i. what is the rate at which customers enter the bank?

ii. what proportion of potential customers actually enter the bank/ Assume that $\lambda = 2$ (in hours) and $\mu_G = 2$. **(8 Marks)**

QUESTION FOUR (15 MARKS)

a) Suppose that it costs $c\mu$ dollars per hour to provide service at a rate μ. Suppose also that we incur a gross profit of A dollars for each customer served. If the system has a capacity N, what service rate μ maximizes our total profit? **(4 Marks)**

b) A single repairperson looks after both machines 1 and 2. Each time it is repaired, machine i stays up for an exponential time with rate λ_i, i = 1, 2. When machine i fails, it requires an exponentially distributed amount of work with rate μ_i to complete its repair. The repairperson will always service machine 1 when it is down. For instance, if machine 1 fails while 2 is being repaired, then the repair person will immediately stop work on machine 2 and start on 1. What proportion of time is machine 2 down? **(4 Marks)**

c) Suppose that customers arrive at a train depot in accordance with a renewal process having a mean interarrival time μ. Whenever there are N customers waiting in the depot, a train leaves.

i. If the depot incurs a cost at the rate of nc dollars per unit time whenever there are n customers waiting, what is the average cost incurred by the depot?

ii. Suppose now that each time a train leaves, the depot incurs a cost of six units. What value of N minimizes the depot's long-run average cost when c = 2, $\mu = 1$? **(7 Marks)**

QUESTION FIVE (15 MARKS)

a) Consider a shoeshine shop consisting of two chairs. Suppose that an entering customer first will go to chair 1. When his work is completed in chair 1, he will go either to chair 2 if that chair is empty or else wait in chair 1 until chair 2 becomes empty. Suppose that a potential customer will enter this shop as long as chair 1 is empty. (Thus, for instance, a potential customer might enter even if there is a customer in chair 2.) If we suppose that potential customers arrive in accordance with a Poisson process at rate λ, and that the service times for the two chairs are independent and have respective exponential rates of μ_1 and μ_2, then

i. what proportion of potential customers enters the system?
ii. what is the mean number of customers in the system?
iii. what is the average amount of time that an entering customer spends in the system? **(6 Marks)**

b) Suppose that a one-celled organism can be in one of two states—either A or B. An individual in state A will change to state B at an exponential rate α; an individual in state B divides into two new individuals of type A at an exponential rate β. Define an appropriate continuous-time Markov chain for a population of such organisms and determine the appropriate parameters for this model. **(5 Marks)**

c) Two machines continually process an unending number of jobs. The time that it takes to process a job on machine 1 is a gamma random variable with parameters n = 4, $\lambda = 2$, whereas the time that it takes to process a job on machine 2 is uniformly distributed between 0 and 4. Approximate the probability that together the two machines can process at least 90 jobs by time t = 100. **(4 Marks)**

QUESTION SIX (15 MARKS)

a) Consider a system of two servers where customers from outside the system arrive at server 1 at a Poisson rate 4 and at server 2 at a Poisson rate 5. The service rates of 1 and 2 are respectively 8 and 10. A customer upon completion of service at server 1 is equally likely to go to server 2 or to leave the system (i.e., $P_{11} = 0$, $P_{12} = {}^{1}/_{2}$); whereas a departure from server 2 will go 25 percent of the time to server 1 and will depart the system otherwise (i.e., $P_{21} = {}^{1}/_{4}$, $P_{22} = 0$). Determine the limiting probabilities, L, and W. **(6 Marks)**

b) The following problem arises in molecular biology. The surface of a bacterium consists of several sites at which foreign molecules—some acceptable and some not—become attached. We consider a particular site and assume that molecules arrive at the site according to a Poisson process with parameter λ. Among these molecules a proportion α is acceptable. Unacceptable molecules stay at the site for a length of time which is exponentially distributed with parameter μ_1, whereas an acceptable molecule remains at the site for an exponential time with rate μ_2. An arriving molecule will become attached only if the site is free of other molecules. What percentage of time is the site occupied with an acceptable (unacceptable) molecule? **(4 Marks)**

c) If the mean-value function of the renewal process $\{N(t), t \geq 0\}$ is given by $m(t) = \frac{t}{2}, t \geq 0$, what is $P\{N(5) = 0\}$? **(5 Marks)**

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