# MAASAI MARA UNIVERSITY

## REGULAR UNIVERSITY EXAMINATIONS
## 2023/2024 ACADEMIC YEAR
## FIRST YEAR SECOND SEMESTER

## SCHOOL OF BUSINESS AND ECONOMICS
## MASTER SCIENCE IN APPLIED STATISTICS

## COURSE CODE: STA 8212/ECS 8204
## COURSE TITLE: TEST OF HYPOTHESIS

**DATE: 2/2/2024** **TIME: 1430-1730 HRS**

**INSTRUCTIONS TO CANDIDATES**

*Answer question **ONE (compulsory)** and any other **TWO** questions*

**Q1 (30 MARKS)**

a) Define the following terms giving an example in each case:
   i. Power of a test (2 marks)
   ii. P value (2 marks)
   iii. Randomized test (2 marks)
   iv. Generalized likelihood ratio test (2 marks)
   v. Uniformly most powerful test (2 marks)

b) State the Newman – Pearson lemma (2 marks)

c) A food processing company packages honey in small glass jars. Each jar is supposed to contain 5 litres of honey. Previous experience suggests that the volume $X$, of a randomly selected jar of the company's honey is normally distributed with a known variance of 2.5. Derive the generalized likelihood ratio test for testing, at a significance level of $\alpha = 0.05$, the null hypothesis $H_0: \mu = 5$ against the alternative hypothesis $H_1: \mu \neq 5$ (4 marks)

d) A study was taken to establish whether there is a difference in the mean salaries between the male and female employees. The salaries in Kshs 1000 for the randomly selected employees by gender are shown below. Test the appropriate hypotheses using $\alpha = 0.05$ significance level.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Male | 65 | 61 | 45 | 49 | 48 | 46 | 61 | 56 | 48 | 53 | 68 | 48 |
| Female | 46 | 50 | 39 | 40 | 53 | 49 | 41 | 53 | 43 | | | |

(3 marks)

e) Let X and Y be two independently distributed random variables with

distributions X ~N $[\mu_1, \sigma_1^2]$ and Y~ $[\mu_2, \sigma_2^2]$ respectively.

Let $x_1$ $x_2$ ….. $x_m$ be a random sample of size m from X and $y_1$, $y_2$, ….., $x_n$ be

Another independent random sample of size n from Y.

Derive the likelihood ratio test for testing $H_0$: $\mu_2 = \mu_1$ against $H_1: \mu_1 \neq \mu_2$,

Assuming $\sigma_1^2 = \sigma_2^2 = \sigma^2$ [8marks]

f) The following statistics were obtained from data drawn from two independent populations X and Y which are normally distributed as follows: X ~N $[\mu_1, \sigma_m^2]$

and Y~ $[\mu_2, \sigma_n^2]$

$\overline{X}$ = 1.02, $\sum_{i=1}^{m}(X_1 - \overline{X})^2$ = 2.44, m = 11

$\overline{Y}$ = 1.66, $\sum_{i=1}^{n}(Y_1 - \overline{Y})^2$ = 4.23, n = 13

Test $H_0$: $\mu_1 = \mu_2$ against $H_1$: $\mu_1 \neq \mu_2$. Use $\propto$ = 5%
[3 marks]

**Q2 (20 MARKS)**

a) Consider the accompanying data on breaking load (kg/25-mm width) for various fabrics in both unabraded condition and abraded condition

| | Fabric | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| U | 36.4 | 55.0 | 51.5 | 38.7 | 43.2 | 48.8 | 25.6 | 49.8 |
| A | 28.5 | 20.0 | 46.0 | 34.5 | 36.5 | 52.5 | 26.5 | 46.5 |

Perform the Wilcoxon signed-rank test to compare the breaking load by fabric type. (8 marks)

b) A physician is interested in the effect of an anesthetic on reaction times. Two groups are compared, one with (A) and one without (B) taking the anesthetic. Subjects had to react on a simple visual stimulus. Reaction times are not normally distributed in this experiment, so data is analyzed with the Mann-Whitney U-Test (Wilcoxon Rank Sum) for ordinal scaled measurements. Are the reaction times different among the two groups?

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Reaction Time | 131 | 135 | 138 | 138 | 139 | 141 | 142 | 142 | 142 | 143 | 144 | 145 | 156 |
| Group | B | A | B | B | A | B | B | A | B | B | A | B | B |
| Reaction Time | 158 | 165 | 167 | 171 | 178 | 191 | 230 | 244 | 245 | 256 | 267 | 268 | 289 |
| Group | A | A | B | A | A | B | B | A | A | A | A | A | A |

(12 marks)

**Q3 (20 MARKS)**

Five teaching methods were used to teach a Statistics course. Students were randomly assigned to the methods and their scores at a CAT are given below:

| **TEACHING** |
|---|

| METHOD | | | | |
|---|---|---|---|---|
| **I** | **II** | **III** | **IV** | **V** |
| 12 | 21 | 26 | 22 | 10 |
| 30 | 21 | 29 | 26 | 13 |
| 15 | 15 | 28 | 17 | 16 |
| 22 | 28 | 22 | 19 | 11 |
| 19 | 11 | 25 | 27 | 30 |
| 14 | 22 | 20 | 26 | 25 |

Are the teaching methods different? Use the parametric approach at $\alpha = 0.05$
(20 marks)

**Q4 (20 MARKS)**

a) A researcher is concerned about the level of knowledge possessed by university students regarding Kenya history. Students completed a high school level Kenya history exam. The course for the students was also recorded. The data is recorded below for 32 students. Test the hypothesis that the level of knowledge of Kenya history is independent of the student's course. Use the non-parametric approach.

| Education | Business | Social Science | Fine Arts |
|---|---|---|---|
| 62 | 72 | 42 | 80 |
| 81 | 49 | 52 | 57 |
| 75 | 63 | 31 | 87 |
| 58 | 68 | 80 | 64 |
| 67 | 39 | 22 | 28 |
| 48 | 79 | 71 | 29 |
| 26 | 40 | 68 | 62 |
| 36 | 15 | 76 | 45 |

(8 marks)

a) Let $x_1, x_2, \ldots X_n$ be a random sample from a normal variable X with
mean $\mu$ and variance $\sigma^2$, where both $\mu$ and $\sigma^2$ are unknown.
Derive the test statistic for testing $H_0$: $\sigma^2 = \sigma_0^2$, against $H_1$: $\sigma^2 \neq \sigma_0^2$
.
Use $\propto$ % [8 marks]

b) A random sample of n = 7 observations from a normal population produced

the following measurements: 4, 0, 6, 3, 3, 2, 5, 9. Do the data provide
sufficient evidence to indicate that $\sigma^2 > 1$? Test using $\propto = 0.05$
[4 marks]

**Q5 (20 MARKS)**

a) (i) Let $(x_1, y_1)$, $(x_2, y_2)$ ..... $(x_n, y_n)$ be a random sample of size n from the bivariate normal population. Derive the test statistic for testing the hypothesis $H_0$: $\ell = 0$ against $H_1$: $\ell \neq 0$, where $\ell$ is the correlation co-efficient between X and Y.
[8 marks]

(ii) Use the test statistic derived in (a) to test the hypothesis $H_0$: $\ell = 0$ against $H_1$: $\ell \neq 0$ at $\propto = 5\%$ if the observations are (33, 24), (60, 34),
(19, 64) (19, 24), (39, 34), assuming this sample is drawn from a bivariate
normal population. [4 marks]

b) Twenty test areas were given different concentrations of fertilizer and the resulting crop yield weighed. The data is given below:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Concentration g/L (x) | 10.2 | 10.6 | 11.2 | 11.6 | 12.2 | 12.6 | 13.2 | 13.6 | 14.2 | 14.6 |
| Crop weight kg (y) | 27.2 | 19.6 | 28.2 | 30.6 | 23.2 | 32.6 | 20.2 | 28.6 | 31.2 | 27.6 |
| Concentration g/L (x) | 15.2 | 15.6 | 16.2 | 16.6 | 17.2 | 17.6 | 18.2 | 18.6 | 19.2 | 19.6 |
| Crop weight kg (y) | 20.2 | 24.6 | 30.2 | 27.6 | 36.2 | 24.6 | 34.2 | 30.6 | 25.2 | 37.6 |

a) Determine the regression equation and perform a test of hypothesis on the slope.
(4 marks)

b) Compute the *Pearson product moment correlation coefficient* and perform a test of hypothesis on it. (4 marks)

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