MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATION
2023/2024 ACADEMIC YEAR
FIRST YEAR FIRST SEMESTER

SCHOOL OF SCIENCE AND INFORMATION SCIENCES
MASTER OF SCIENCE (APPLIED STATISTICS)

COURSE CODE: STA 8104
COURSE TITLE: STOCHASTIC PROCESSES 1

DATE: 1/2/2024 **TIME: 0830-1030 HRS**

INSTRUCTIONS TO CANDIDATES

i. Question **ONE** is compulsory
ii. Answer any other **TWO** questions

QUESTION ONE (20 MARKS)

a. Define the following terms as used in stochastic process

i. A state space (1mark)
ii. A generating function (1mark)
iii. Discrete time process (1mark)
iv. A stochastic process {X(t), x□T} (1mark)
v. An ergodic (1mark)

b. If q_k=2 for all k such that a_0=a_1=….q_k=2, find the generation function A(s) (1mark)

c. Suppose that customers arrive a bank according to a Poisson process with a mean rate of a, per minute. Then the number of customer's N (t) arriving in an interval of duration t minutes follows Poisson distribution with mean at. If the rate of arrival in 3 per minute, then in an arrival of 2 minutes.

Find the probability the number of customers arriving is

i. Exactly 4 (1mark)
ii. Greater than 4 (1mark)
iii. Less than 4 (1mark)

d.i) When is a stochastic process said to be a stationery? (2marks)

ii.Let x_n, n□1 be uncorrelated random variable with mean 0 and variance 1.

Show that $c(n, m)=cov\ (x_n, x_m)$ (3marks)

e. Suppose that (f_n n =1,2….)(b_n n = 0,1,2,)are two sequences of real numbers such that $f_n \geq 0, f = \sum f_n$□∞ *and* $b_n \geq o$, $b = \sum b_n$□∞

Define a sequence (v_n, n = 0, 1…) by the convolution (4marks)

f. State the condition when a state is to be persistent and transient (2marks)

QUESTION TWO (20 MARKS)

Classify the states of the following $p = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{1} & 0 \end{pmatrix}$ (20marks)

QUESSTION THREE (20 MARKS)

A stochastic process x (t) is a Poisson process. The probability that k events occur between t and t + h given that n events occurred by exponential t is given by

$$P_k (h) = p_r [N (h) = k/N(t) = n] = \begin{Bmatrix} \lambda (h) + 0(h), k = 1 \\ 0(h), k \geq 2 \\ 1 - \lambda h + 0(h), k = 0 \end{Bmatrix}$$

Required:

Show that differential equation of the Poisson processes are given by

$P^1{}_n(t) = \lambda(p_n(t)\text{-}p_n\text{-}1\ (t)), n \geq 1$ (15marks)

$P^1{}_u(t) = \lambda p_0(t)$ (5marks)

QUESTION FOUR (20 MARKS

a. Given that $p(x = k) = \begin{Bmatrix} c^{-\lambda}\lambda k, k = 0,1,2 \ldots. \\ k! \\ 0\ otherwise \end{Bmatrix}$

Find the probability generating function of random variable x (10marks)

b. Consider the process: $x(t) = A_1 + A_2(t)$ where A_1A_2 are independent random variable with: $E(A_1) = a_i$, = var $(A_i) = a_1^2$, i = 1 ,2 have $m(t)= a_1 + a_2 (t)$

Show that the process is evolutionary (10marks)

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