# MAASAI MARA UNIVERSITY

## REGULAR UNIVERSITY EXAMINATIONS
## 2023/2024 ACADEMIC YEAR
## FIRST YEAR FIRST SEMESTER

## SCHOOL OF PURE APPLIED AND HEALTH SCIENCES
## MASTER OF SCIENCE IN PURE MATHEMATICS

## COURSE CODE: MAT 8104
## COURSE TITLE: TOPOLOGY I

**DATE:** **DURATION:**

**INSTRUCTIONS TO CANDIDATES**

Answer Question **ONE** and any other **TWO** questions

*This paper consists of **THREE** printed pages. Please turn over.*

**QUESTION ONE (30 MARKS)**

**a)** Let $(X,d)$ be a metric space, show that $\forall x,y,z \in X$, $|d(x,z)-d(y,z)| \leq d(x,y)$. **(3 marks)**

**b)** Let $(X,T)$ be a topological space and let $A \subset X$, show that $\overline{A^c} = (\overset{0}{A})^c$. **(3 marks)**

**c)** Let f be a continuous function from a topological space X into $\mathbb{R}$. Let a be a real number and let $A = \{x \in X : f(x) = a\}$, verify that a is closed in X. **(3 marks)**

**d)** In the usual topology of $\mathbb{R}$, show that $A = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$ is not compact using open covers. **(4 marks)**

**e)** Using only the definition of path connected set, show that $\mathbb{R}^*$ is not path connected. **(4 marks)**

**f)** Show that the plane $\mathbb{R}^2$ with the usual topology satisfies second axiom of countability. **(3 marks)**

**g)** Show that the class $C(X,\mathbb{R})$ of all real valued continuous functions on a completely regular $T_1$-space $X$ separates points. **(4 marks)**

**h)** Let X be a Hausdorff space, show that every convergent sequence in X has a unique limit. **(6 marks)**

**QUESTION TWO (15 MARKS)**

**a)** Let $\mathbb{R}$ be endowed with its standard topology, find the closure of $\mathbb{Q}$ and $\mathbb{R} \setminus \mathbb{Q}$. **(3 marks)**

**b)** Define a Hausdorff property hence show that Hausdorff is a topological property. **(5 marks)**

**c)** Let X and Y be two topological spaces, show that if $X \times Y$ is compact, then X and Y are also compact **(4 marks)**

**d)** Show that any subspace $(Y, \tau_Y)$ of a first countable space $(X, \tau)$ is also first countable. **(3 marks)**

**QUESTION THREE (15 MARKS)**

**a)** Let (X,d) be a metric space. Let $r > 0$ and let $x \in X$. We denote the open ball in X of centre x and radius r by B(x,R) while the closed ball in X of centre x and radius r is denoted by $B_c(x,r)$. **(3 marks)**

**b)** Let X be a non-empty set. Let $a \in X$ be fixed and set $T = \{\phi\} \cup \{U \subset X : a \in U\}$

i. Is T Hausdorff **(2 marks)**

ii. Find $\{a\}'$ **(2 marks)**

iii. Show that a completely regular space is regular. **(4 marks)**

**c)** Show that a discrete space X is separable if and only if X is countable. **(5 marks)**

**QUESTION FOUR (15 MARKS)**

**a)** Let X be a non-empty set, define a map on $X \times X$ by $d(x,y) = \begin{cases} 0, & x = y \\ 1, & x \neq y \end{cases}$

i. Let r>0 and let $x \in X$ , find the open ball $B(x,r)$ and the close ball $B_c(x,r)$. **(3 marks)**

ii. Find the sphere $S(x,r)$. **(2 marks)**

**b)** Show that every subset in a discrete metric space is open. **(2 marks)**

**c)** Deduce that every subset in a discrete metric space is closed. **(1 mark)**

**d)** Let A be any subset of a second countable space X, prove that if $\mathfrak{H}$ is an open cover of A, then $\mathfrak{H}$ is reducible to a countable cover. **(4 marks)**

**e)** Let τ be the coffinite topology on any set X. Show that $(X,\tau)$ is separable. **(3 marks)**