MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS

2023/2024

SCHOOL OF BUSINESS AND ECONOMICS

MASTER OF SCIENCE IN ECONOMICS AND STATISTICS

SECOND YEAR FIRST SEMESTER

COURSE CODE: ECS 8205

COURSE TITLE: STOCHASTIC PROCESSES

DATE: **TIME:**

INSTRUCTIONS:
Attempt any ***Three*** Questions

Question One

a. State the Markovian property for a discrete stochastic process $\{X_n : n = 0,1,2,\ldots\}$ with state space S. **(2 marks)**

b. A music instrument store is open every day of the week except Monday. During that day, if the inventory count is below 3, more instruments are ordered, so that by Tuesday morning there are 7 instruments in stock. If 3 or more instruments are in stock, then no action is taken. The number of instruments sold during the business days is a Poisson random variable with a mean of 4. Any demand that cannot be satisfied is lost.

   i. Argue that the inventory each Tuesday morning can be modelled as a Markov chain. Find its state space and the one-step transition probability matrix. **(3 marks)**

   ii. Suppose one week there are 7 instruments in stock on Tuesday morning. Compute the probability that there will be 7 instruments in stock also on each of the three subsequent Tuesday mornings. **(3 marks)**

c. The Chapman – Kolmogorov equations asserts that for positive integers m and n, and one step transition probability matrix P.

   $P^{(n+m)} = P^{(n)}.P^{(m)}$

   Prove. **(4 marks)**

d. The transition of customers between mobile networks in the country (S={Safaricom, Airtel, Telcom}) is known to be governed by a single step transition probability matrix $P = \begin{bmatrix} 0.92 & 0.05 & 0.03 \\ 0.6 & 0.35 & 0.05 \\ 0.8 & 0.05 & 0.15 \end{bmatrix}$. If the current mobile network market share is 90% safaricom, 8% Airtel and 2% Telcom. Determine

   i. The market shares two years from now. **(2 marks)**

   ii. The market shares in the long run. **(4 marks)**

   iii. $\Pr(X_0 = 1, X_1 = 1, X_3 = 2)$. **(2 marks)**

Question Two

a. Give the four properties of a Poisson process $\{N(t), t \geq 0\}$ with rate λ. **(2 marks)**

b. Let T_n be the interarrival time between two consecutive events of a Poisson process $\{N(t), t \geq 0\}$ with rate. Show that T_n is exponentially distributed with mean $1/\lambda$. **(5 marks)**

c. Let S_n be the waiting time until an event n takes place in a Poisson process $\{N(t), t \geq 0\}$ with rate λ. Determine the distribution of S_n. **(3 marks)**

d. Tour buses arrive at a mall which has a restaurant, bringing 100 tourists each. The time that elapse between consecutive arrivals are independent and exponentially distributed with mean of 20 minutes.
   i. Determine the distribution of arrival rate of buses at the mall. **(2 marks)**
   ii. Determine the mean and variance of the waiting time for the arrival of the 5th bus (S_5). **(3 marks)**
   iii. Determine the expected number of tourist at the mall between 11:30 am and 2:30 pm. **(2 marks)**
   iv. Determine the probability that between 11:30 am and 2:30 pm, more than 400 tourists arrive at the mall. **(3 marks)**

Question Three

a. Let $\{X(t), t \geq 0\}$ be a birth-and-death process with state space $S = \{0,1,2,\ldots\}$. If given that the time to transition from state n to state $n+1$ is exponentially distributed with mean $1/\lambda_n$ and the time to transition from state n to state $n-1$ is exponentially distributed with mean $1/\mu_n$. Show that the transition probabilities are $p_{0,1} = 1$, $p_{n,n+1} = \dfrac{\lambda_n}{\lambda_n + \mu_n}$ and $p_{n,n-1} = \dfrac{\mu_n}{\lambda_n + \mu_n}$. **(5 marks)**

b. An $M/M/1$ queue is a birth-and-death process with $\lambda_n = \lambda$ and $\mu_n = \mu$.
   i. Determine the Kolmogorov forward equation for the process. **(3 marks)**
   ii. Using the Kolmogorov forward balance equation. Show that P_n is geometric distributed with probability of success $p = 1 - \dfrac{\lambda}{\mu}$. **(5 marks)**
   iii. Show that the average number of customers in the system in the long run is given by $\dfrac{\lambda}{\mu - \lambda}$. **(3 marks)**
   iv. If $\lambda = 2$ and $\mu = 4$, determine the probability that a customer will wait more than 5 minutes in the system to be served. **(3 marks)**
   v. What will happen to the queue if $\lambda > \mu$? Explain. **(1 marks)**

Question Four

a. Give the properties that a stochastic process $\{B(t), t \geq 0\}$ must satisfy to be considered as a standard Brownian motion. **(4 marks)**

b. Let $\{B(t), t \geq 0\}$ be a standard Brownian motion, show that for $s, t \geq 0$ $\operatorname{cov}(B(s), B(t)) = \min(s,t)$. **(4 marks)**

c. Let $\{B(t), t \geq 0\}$ and $\{W(t), t \geq 0\}$ be two independent standard Brownian motions. Show that the process $X(t) = \dfrac{B(t) + W(t)}{\sqrt{2}}$ is also a standard Brownian motion.

(6 marks)

d. Let $\{W(t), t \geq 0\}$ be a standard Brownian motion with probability density function $\mu(x,t)$. Show that the process satisfy the one dimension heat equation $\dfrac{\partial \mu(x.t)}{\partial t} = \dfrac{1}{2}\dfrac{\partial^2 \mu(x,t)}{\partial x^2}$.

(6 marks)