MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2023/ 2024 ACADEMIC YEAR
SECOND YEAR FIRST SEMESTER

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES
DEGREE IN APPLIED STATISTICS WITH COMPUTING

COURSE CODE: STA 2112-1

COURSE TITLE: MATHEMATICAL STATISTICS I

DATE: 4/6/2023 **TIME: 0830-1030 HRS**

INSTRUCTIONS TO CANDIDATES

Answer Question ONE and any other TWO questions

Question one (20 marks)

a) Suppose that X is a random variable with probability density

$$f(x)=\begin{cases}1 & 0<x<1\\ 0 & \text{elsewhere}\end{cases}$$

Find the density function of Y=-2lnX **(5 marks)**

b) The joint probability function of X and Y is given by

$$f(x,y)=\begin{cases}kxy & 0<x<1,\ 0<y<1\\ 0 & \text{elsewhere}\end{cases}$$ Find the:

(i) Value of k that makes this a probability density function
(ii) Marginals of X and Y
(iii)The conditional distribution of X given Y
(iv) $p(x \le 0.5, y < 0.75$
(v) $p(x \le 1/2 / Y \ge 3/4)$ **(10 marks)**

c) The joint probability of X and Y is given by

$$f(x,y)=\begin{cases}12xy(1-y) & 0<x<1,\ 0<y<1\\ 0 & \text{elsewhere}\end{cases}$$

Determine if X and Y are independent or not **(5 marks)**

Question two (15 marks)

a) Let X and Y be two random variables with joint PDF given as

$$f(x,y)=\begin{cases}e^{-y} & 0<x<y<\infty\\ 0 & \text{elsewhere}\end{cases}$$

(i) Determine the mgf of X and Y
(ii) Find the mgf of X
(iii)Determine the covariance between X and Y **(12 marks)**

b) If X and Y are joint random variables with mgf, show that the mean of X is given by `

$$E(x) = \frac{dM(t_1, t_2)}{dt_1 / t_1 = t_2 = 0}$$ **(3 marks)**

Question three (15 marks)

Let W and H have joint density distribution given by

$$f(w,h) = \begin{cases} \left(\frac{2}{3}\right)^{w+h}\left(\frac{1}{3}\right)^{w-h} & \text{(w,h)=(0,0), (0,1),(1,0),(1,1)} \\ 0 & \text{elsewhere} \end{cases}$$

Find the joint pdf of
$X = W - H$ *and* $Y = W + H$

(15 marks)

Question four (15 marks)

Let $Y_1 < Y_2 < Y_3 < Y_4 < Y_5$ denote order statistics of size 5 from a distribution having a pdf

$$f(x) = \begin{cases} e^{-x} & 0<x<\infty \\ 0 & \text{elsewhere} \end{cases}$$

Find

i) The joint pdf of Y_2 *and* Y_4

ii) The pdf of Y_3

iii) The joint pdf of $X = Y_2$ *and* $Z = Y_4 - Y_2$

/END/