MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR
SECOND YEAR FIRST SEMESTER
SCHOOL OF SCIENCE
B.SC MATHEMATICS
B.SC PHYSICS

MAIN CAMPUS

COURSE CODE: MAT 2216-1
COURSE TITLE: ADVANCED CALCULUS

DATE: 21st April, 2023 **TIME: 1400-1600 HRS (2 hours)**

INSTRUCTIONS TO CANDIDATES

QUESTION ONE IS COMPULSORY then answer ANY **OTHER TWO QUESTIONS**

DO NOT WRITE ANYTHING ON THIS QUESTION PAPER

This paper consists of 4 printed pages.

QUESTION ONE (20 MARKS)

a) Define the improper integral $\int_a^b f(x)dx$ for each of the following cases

i. f has an infinite discontinuity at a. (1 Marks)

ii. f has an infinite discontinuity at b. (1 Marks)

iii. f has an infinite discontinuity at c where $a < c < b$. (1 Marks)

b) Determine whether the integral

$$\int_{-\infty}^{0} xe^x dx$$

is convergent or divergent. (3 Marks)

c) If $Z = 2x^2 - 3xy + 4y^2$. Find

i. Z_x (1 Marks)

ii. Z_y (1 Marks)

iii. Z_{xy} (1 Marks)

iv. Z_{yx} (1 Marks)

v. Z_{yy} (1 Marks)

d) Show that

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2 + y^4}$$

does not exist. (3 Marks)

e) Find the sum of the series of the series (2 Marks)

$$\sum_{n=0}^{\infty} x^n , where\ |x| < 1$$

f) Find the first three non-zero terms in Maclaurin series for

i. $e^x \sin x$ (2 Marks)

ii. $\tan x$ (2 Marks)

QUESTION TWO (15 MARKS)

a) Find the radius of convergence and interval of convergence of the series (5 Marks)

$$\sum_{n=0}^{\infty}\left(\frac{(-3)^n x^n}{\sqrt{n+1}}\right)$$

b) Show that if f has a power series representation (expansion) at a, that is, if

$$f(x)=\sum_{n=0}^{\infty} C_n (x-a)^n, \quad |x-a|<R$$

then its coefficients are given by the formula (4 Marks)

$$C_n=\frac{f^n(a)}{n!}$$

c) Obtain Maclaurin series expansion for $f(x)=e^{-2x}$. (3 Marks)

d) By making use of the Taylor series expansion. Find the value of $\sin 62^{\circ}$ correct to 5 d.p.

(3 Marks)

QUESTION THREE (15 MARKS)

a) Evaluate $\int y^2\,dx + x\,dy$, where

i. $C=C_1$ is the line segment from $(-5,-3)$ to $(0,2)$. (2 Marks)

ii. $C=C_2$ is the arc of the parabola from $(-5,-3)$ to $(0,2)$. (2 Marks)

b) State the Green's theorem. (2 Marks)

c) Compute the surface integral $\iint x^2\,dS$, where S is the unit sphere $x^2+y^2+z^2=1$

(4 Marks)

d) State the divergence theorem. (2 Marks)

e) Find the flux of the vector field $\boldsymbol{F}(x,y,z)=z\,\boldsymbol{i}+y\,\boldsymbol{j}+x\,\boldsymbol{k}$ over the unit sphere

$x^2+y^2+z^2=1$ (3 Marks)

QUESTION FOUR (15 MARKS)

a) Find $\lim_{n\to\infty} \frac{n}{n+1}$ (2 Marks)

b) Investigate the convergence or divergence of $\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt[3]{8\,n^2-5n}}\right)$ (4 Marks)

c) Use the ratio test to investigate the convergence of $a_n = \frac{2\,n-1}{4^n\,2^n\,n!}$ (4 Marks)

d) A drug is administered to a patient at the same time every day. Suppose the concentration of the drug is C_n (measured in mg/mL) after the injection on the nth day. Before the injection the next day, only 30% of the drug remains in the bloodstream and the daily dose raises the concentration by $0.2\,mg/mL$.

i. Find the concentration after three days. (2 Marks)

ii. What is the concentration after the nth dose? (2 Marks)

iii. What is the limiting concentration? (1 Mark)

QUESTION FIVE (15 MARKS)

a) Find the local maximum and minimum values and the saddle points of

$f(x,y) = x^4 + y^4 - 4xy + 1$ (4 Marks)

b) A rectangular box without a lid is to be made from $12m^2$ of cardboard. Find the maximum volume of such a box. (4 Marks)

c) Determine the set of points at which the function is continuous (4 Marks)

$$f(x,y) = \begin{cases} \frac{xy}{x^2+xy+y^2}, & if\ (x,y) \neq (0,0) \\ 0, & if\ (x,y) = (0,0) \end{cases}$$

d) If $u = x^4y + y^2z^3$ where $x = rse^t, y = rs^2e^{-t}, z = r^2s\sin t$. By use of a tree diagram, find the value of $\frac{\partial u}{\partial s}$ when $r = 2, s = 1, t = 0$. (4 Marks)

THE END
