Appendix A

Tables

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Table A.1 Upper Percentiles for $\sqrt{b_1}$

$$\sqrt{b_1} = \frac{\sqrt{n}\sum_{i=1}^{n}(y_i - \overline{y})^3}{\left[\sum_{i=1}^{n}(y_i - \overline{y})^2\right]^{3/2}}$$

The sampling distribution of $\sqrt{b_1}$ is symmetric about zero, and the lower percentage points corresponding to negative skewness are given by the negative of the table values. Reject the hypothesis of normality if $\sqrt{b_1}$ is greater than the table value or less than the negative of the table value.

	Upper Percentiles					
n	10	5	2.5	1	0.5	0.1
4	.831	.987	1.070	1.120	1.137	1.151
5	.821	1.049	1.207	1.337	1.396	1.464
6	.795	1.042	1.239	1.429	1.531	1.671
7	.782	1.018	1.230	1.457	1.589	1.797
8	.765	.998	1.208	1.452	1.605	1.866
9	.746	.977	1.184	1.433	1.598	1.898
10	.728	.954	1.159	1.407	1.578	1.906
11	.710	.931	1.134	1.381	1.553	1.899
12	.693	.910	1.109	1.353	1.526	1.882
13	.677	.890	1.085	1.325	1.497	1.859
14	.662	.870	1.061	1.298	1.468	1.832
15	.648	.851	1.039	1.272	1.440	1.803
16	.635	.834	1.018	1.247	1.412	1.773
17	.622	.817	.997	1.222	1.385	1.744
18	.610	.801	.978	1.199	1.359	1.714
19	.599	.786	.960	1.176	1.334	1.685
20	.588	.772	.942	1.155	1.310	1.657
21	.578	.758	.925	1.134	1.287	1.628
22	.568	.746	.909	1.114	1.265	1.602
23	.559	.733	.894	1.096	1.243	1.575
24	.550	.722	.880	1.078	1.223	1.550
25	.542	.710	.866	1.060	1.203	1.526

Table A.2 Coefficients for Transforming $\sqrt{b_1}$ to a Standard Normal

n	δ	$1/\lambda$	n	δ	$1/\lambda$	n	δ	$1/\lambda$
8	5.563	0.3030	41	3.073	0.9653	98	3.913	1.1006
9	4.260	0.4080	42	3.087	0.9702	100	3.940	1.1028
10	3.734	0.4794	43	3.102	0.9750	105	4.009	1.1080
11	3.447	0.5339	44	3.117	0.9795	110	4.076	1.1128
12	3.270	0.5781	45	3.131	0.9840	115	4.142	1.1172
13	3.151	0.6153	46	3.146	0.9882	120	4.207	1.1212
14	3.069	0.6473	47	3.161	0.9923	125	4.272	1.1250
15	3.010	0.6753	48	3.176	0.9963	130	4.336	1.1285
16	2.968	0.7001	49	3.192	1.0001	135	4.398	1.1318
17	2.937	0.7224	50	3.207	1.0038	140	4.460	1.1348
18	2.915	0.7426	52	3.237	1.0108	145	4.521	1.1377
19	2.900	0.7610	54	3.268	1.0174	150	4.582	1.1403
20	2.890	0.7779	56	3.298	1.0235	155	4.641	1.1428
21	2.884	0.7934	58	3.329	1.0293	160	4.700	1.1452
22	2.882	0.8078	60	3.359	1.0348	165	4.758	1.1474
23	2.882	0.8211	62	3.389	1.0400	170	4.816	1.1496
24	2.884	0.8336	64	3.420	1.0449	175	4.873	1.1516
25	2.889	0.8452	66	3.450	1.0495	180	4.929	1.1535
26	2.895	0.8561	68	3.480	1.0540	185	1.985	1.1553
27	2.902	0.8664	70	3.510	1.0581	190	5.040	1.1570
28	2.910	0.8760	72	3.540	1.0621	195	5.094	1.1586
29	2.920	0.8851	74	3.569	1.0659	200	5.148	1.1602
30	2.930	0.8938	76	3.599	1.0695	205	5.202	1.1616
31	2.941	0.9020	78	3.628	1.0730	210	5.255	1.1631
32	2.952	0.9097	80	3.657	1.0763	215	5.307	1.1644
33	2.964	0.9171	82	3.686	1.0795	220	5.359	1.1657
34	2.977	0.9241	84	3.715	1.0825	225	5.410	1.1669
35	2.990	0.9308	86	3.744	1.0854	230	5.461	1.1681
36	3.003	0.9372	88	3.772	1.0882	235	5.511	1.1693
37	3.016	0.9433	90	3.801	1.0909	240	5.561	1.1704
38	3.030	0.9492	92	3.829	1.0934	245	5.611	1.1714
39	3.044	0.9548	94	3.857	1.0959	250	5.660	1.1724
40	3.058	0.9601	96	3.885	1.0983			

Values of δ and $1/\lambda$ are such that $g(\sqrt{b_1}) = \delta \sinh^{-1}(\sqrt{b_1}/\lambda)$ is approximately $N(0, 1)$.

Table A.3 Percentiles for b_2

Upper and lower percentiles for

$$b_2 = \frac{n\sum_{i=1}^{n}(y_i - \bar{y})^4}{\left[\sum_{i=1}^{n}(y_i - \bar{y})^2\right]^2},$$

the sample coefficient of kurtosis. Reject the hypothesis of normality if b_2 is greater than an upper percentile or less than a lower percentile.

Sample	Percentiles											
Size	1	2	2.5	5	10	20	80	90	95	97.5	98	99
7	1.25	1.30	1.34	1.41	1.53	1.70	2.78	3.20	3.55	3.85	3.93	4.23
8	1.31	1.37	1.40	1.46	1.58	1.75	2.84	3.31	3.70	4.09	4.20	4.53
9	1.35	1.42	1.45	1.53	1.63	1.80	2.98	3.43	3.86	4.28	4.41	4.82
10	1.39	1.45	1.49	1.56	1.68	1.85	3.01	3.53	3.95	4.40	4.55	5.00
12	1.46	1.52	1.56	1.64	1.76	1.93	3.06	3.55	4.05	4.56	4.73	5.20
15	1.55	1.61	1.64	1.72	1.84	2.01	3.13	3.62	4.13	4.66	4.85	5.30
20	1.65	1.71	1.74	1.82	1.95	2.13	3.21	3.68	4.17	4.68	4.87	5.36
25	1.72	1.79	1.83	1.91	2.03	2.20	3.23	3.68	4.16	4.65	4.82	5.30
30	1.79	1.86	1.90	1.98	2.10	2.26	3.25	3.68	4.11	4.59	4.75	5.21
35	1.84	1.91	1.95	2.03	2.14	2.31	3.27	3.68	4.10	4.53	4.68	5.13
40	1.89	1.96	1.98	2.07	2.19	2.34	3.28	3.67	4.06	4.46	4.61	5.04
45	1.93	2.00	2.03	2.11	2.22	2.37	3.28	3.65	4.00	4.39	4.52	4.94
50	1.95	2.03	2.06	2.15	2.25	2.41	3.28	3.62	3.99	4.33	4.45	4.88

Table A.4 Percentiles for D'Agostino's Test for Normality

Upper and lower percentiles for the statistic

$$Y = \frac{\sqrt{n}[D - (2\sqrt{\pi})^{-1}]}{.02998598},$$

where

$$D = \frac{\sum_{i=1}^{n} \left[i - \frac{1}{2}(n+1)\right] y_{(i)}}{\sqrt{n^3 \sum_{i=1}^{n} (y_i - \bar{y})^2}}$$

and the observations $y_1, y_2, \ldots, y_n$ are ordered as $y_{(1)} \leq y_{(2)} \leq \cdots \leq y_{(n)}$. Reject the hypothesis of normality if Y is greater than an upper percentile or less than a lower percentile.

	Percentiles of Y									
n	.5	1.0	2.5	5	10	90	95	97.5	99	99.5
10	−4.66	−4.06	−3.25	−2.62	−1.99	.149	.235	.299	.356	.385
12	−4.63	−4.02	−3.20	−2.58	−1.94	.237	.329	.381	.440	.479
14	−4.57	−3.97	−3.16	−2.53	−1.90	.308	.399	.460	.515	.555
16	−4.52	−3.92	−3.12	−2.50	−1.87	.367	.459	.526	.587	.613
18	−4.47	−3.87	−3.08	−2.47	−1.85	.417	.515	.574	.636	.667
20	−4.41	−3.83	−3.04	−2.44	−1.82	.460	.565	.628	.690	.720
22	−4.36	−3.78	−3.01	−2.41	−1.81	.497	.609	.677	.744	.775
24	−4.32	−3.75	−2.98	−2.39	−1.79	.530	.648	.720	.783	.822
26	−4.27	−3.71	−2.96	−2.37	−1.77	.559	.682	.760	.827	.867
28	−4.23	−3.68	−2.93	−2.35	−1.76	.586	.714	.797	.868	.910
30	−4.19	−3.64	−2.91	−2.33	−1.75	.610	.743	.830	.906	.941
32	−4.16	−3.61	−2.88	−2.32	−1.73	.631	.770	.862	.942	.983
34	−4.12	−3.59	−2.86	−2.30	−1.72	.651	.794	.891	.975	1.02
36	−4.09	−3.56	−2.85	−2.29	−1.71	.669	.816	.917	1.00	1.05
38	−4.06	−3.54	−2.83	−2.28	−1.70	.686	.837	.941	1.03	1.08
40	−4.03	−3.51	−2.81	−2.26	−1.70	.702	.857	.964	1.06	1.11
42	−4.00	−3.49	−2.80	−2.25	−1.69	.716	.875	.986	1.09	1.14
44	−3.98	−3.47	−2.78	−2.24	−1.68	.730	.892	1.01	1.11	1.17
46	−3.95	−3.45	−2.77	−2.23	−1.67	.742	.908	1.02	1.13	1.19
48	−3.93	−3.43	−2.75	−2.22	−1.67	.754	.923	1.04	1.15	1.22
50	−3.91	−3.41	−2.74	−2.21	−1.66	.765	.937	1.06	1.18	1.24
60	−3.81	−3.34	−2.68	−2.17	−1.64	.812	.997	1.13	1.26	1.34
70	−3.73	−3.27	−2.64	−2.14	−1.61	.849	1.05	1.19	1.33	1.42
80	−3.67	−3.22	−2.60	−2.11	−1.59	.878	1.08	1.24	1.39	1.48
90	−3.61	−3.17	−2.57	−2.09	−1.58	.902	1.12	1.28	1.44	1.54
100	−3.57	−3.14	−2.54	−2.07	−1.57	.923	1.14	1.31	1.48	1.59
150	−3.409	−3.009	−2.452	−2.004	−1.520	.990	1.233	1.423	1.623	1.746
200	−3.302	−2.922	−2.391	−1.960	−1.491	1.032	1.290	1.496	1.715	1.853
250	−3.227	−2.861	−2.348	−1.926	−1.471	1.060	1.328	1.545	1.779	1.927

Table A.5 Upper Percentiles for $b_{1,p}$ and Upper and Lower Percentiles for $b_{2,p}$

Reject the hypothesis of multivariate normality if $b_{1,p}$ is greater than table value. Reject if $b_{2,p}$ is greater than upper percentile or if $b_{2,p}$ is less than lower percentile. The statistics $b_{1,p}$ and $b_{2,p}$ are defined in Section 4.4.2.

$p = 2$, Upper Percentiles for $b_{1,p}$							$p = 2$, Upper and Lower Percentiles for $b_{2,p}$								
	Percentiles							Percentiles							
n	90	92.5	95	97.5	99	99.9	n	1	2.5	5	10	90	95	97.5	99
10	2.994	3.263	3.694	4.294	5.194	6.994	10	4.580	4.722	4.887	5.057	8.606	9.203	9.781	10.378
12	2.681	2.944	3.319	3.931	4.938	6.744	12	4.732	4.899	5.053	5.232	8.947	9.593	10.150	10.881
14	2.419	2.669	3.031	3.619	4.581	6.419	14	4.842	5.015	5.179	5.358	9.162	9.769	10.375	11.159
16	2.219	2.444	2.775	3.337	4.231	6.062	16	4.977	5.149	5.318	5.482	9.331	9.941	10.562	11.387
18	2.050	2.256	2.556	3.100	3.962	5.737	18	5.045	5.219	5.382	5.555	9.403	10.005	10.628	11.478
20	1.894	2.081	2.356	2.881	3.669	5.425	20	5.175	5.262	5.533	5.717	9.469	10.114	10.691	11.609
25	1.581	1.744	1.969	2.438	3.106	4.719	25	5.351	5.525	5.689	5.871	9.503	10.159	10.584	11.628
30	1.363	1.513	1.687	2.094	2.681	4.238	30	5.518	5.692	5.855	6.038	9.516	10.156	10.556	11.594
40	1.050	1.181	1.319	1.606	2.087	3.369	40	5.703	5.871	6.139	6.229	9.497	10.109	10.563	11.453
50	.862	.969	1.069	1.306	1.744	2.706	50	5.909	6.083	6.239	6.403	9.453	9.987	10.372	11.181
60	.731	.819	.906	1.094	1.444	2.200	60	6.015	6.189	6.335	6.505	9.401	9.889	10.250	10.994
70	.631	.725	.794	.937	1.244	1.863	70	6.139	6.290	6.437	6.602	9.356	9.781	10.106	10.753
80	.544	.637	.694	.812	1.056	1.587	80	6.223	6.372	6.539	6.683	9.309	9.694	9.981	10.537
90	.487	.569	.638	.725	.919	1.400	90	6.332	6.475	6.622	6.749	9.256	9.688	9.885	10.325
100	.438	.506	.581	.656	.831	1.231	100	6.389	6.521	6.665	6.793	9.210	9.556	9.806	10.188
150	.281	.344	.400	.444	.531	.794	150	6.615	6.749	6.858	6.972	9.027	9.300	9.475	10.253
200	.219	.269	.300	.331	.394	.569	200	6.761	6.889	6.979	7.083	8.919	9.141	9.269	9.506
300	.144	.169	.209	.225	.256	.369	300	6.949	7.052	7.142	7.245	8.776	8.916	9.031	9.219
400	.116	.129	.141	.166	.197	.275	400	7.079	7.171	7.252	7.342	8.664	8.787	8.917	9.061
600	.077	.085	.094	.110	.131	.183	600	7.232	7.295	7.369	7.464	8.547	8.647	8.749	8.874

(continued)

Table A.5 (*Continued*)

$p = 2$,
Upper Percentiles for $b_{1,p}$

	Percentiles					
n	90	92.5	95	97.5	99	99.9
800	.058	.064	.071	.083	.099	.137
1000	.046	.051	.057	.066	.079	.110
1500	.031	.034	.038	.044	.053	.074
2500	.019	.021	.023	.027	.032	.044
3000	.016	.017	.019	.022	.027	.037
4000	.012	.013	.014	.017	.020	.028
5000	.009	.010	.011	.013	.016	.022

$p = 2$,
Upper and Lower Percentiles for $b_{2,p}$

	Percentiles							
n	1	2.5	5	10	90	95	97.5	99
800	7.304	7.372	7.451	7.536	8.472	8.562	8.641	8.747
1000	7.367	7.433	7.504	7.585	8.419	8.497	8.569	8.656
1500	7.460	7.537	7.595	7.661	8.339	8.405	8.463	8.532
2000	7.535	7.599	7.649	7.707	8.293	8.351	8.401	8.461
2500	7.588	7.641	7.686	7.738	8.262	8.314	8.359	8.412
3000	7.624	7.673	7.714	7.760	8.240	8.286	8.327	8.376
4000	7.674	7.716	7.752	7.793	8.207	8.248	8.284	8.326
5000	7.709	7.746	7.778	7.714	8.186	8.222	8.254	8.291

$p = 3$,
Upper Percentiles for $b_{1,p}$

	Percentiles					
n	90	92.5	95	97.5	99	99.9
10	6.0	6.5	6.9	7.7	8.8	11.5
12	5.5	5.9	6.4	7.1	8.1	10.5
14	5.0	5.4	5.9	6.5	7.4	9.7
16	4.6	4.9	5.4	6.1	6.8	8.9
18	4.2	4.6	5.1	5.6	6.4	8.3
20	3.9	4.2	4.7	5.3	6.0	7.7
25	3.3	3.5	3.9	4.5	5.2	6.5
30	2.8	3.0	3.3	3.9	4.4	5.6
40	2.2	2.4	2.7	3.0	3.5	4.2
50	1.7	1.9	2.2	2.4	2.8	3.4

$p = 3$,
Upper and Lower Percentiles for $b_{2,p}$

	Percentiles							
n	1	2.5	5	10	90	95	97.5	99
10	10.0	10.2	10.4	10.7	14.0	14.4	15.0	15.6
12	10.2	10.4	10.7	11.0	14.7	15.2	15.9	16.4
14	10.4	10.6	10.9	11.3	15.1	15.8	16.5	17.1
16	10.5	10.8	11.1	11.5	15.4	16.1	16.8	17.5
18	10.7	11.0	11.3	11.6	15.5	16.4	17.1	17.8
20	10.8	11.1	11.4	11.8	15.7	16.5	17.2	18.0
25	11.1	11.4	11.8	12.1	15.9	16.7	17.4	18.2
30	11.3	11.6	12.0	12.3	16.0	16.7	17.5	18.3
40	11.7	12.0	12.4	12.7	16.1	16.7	17.4	18.2
50	11.9	12.3	12.6	12.9	16.1	16.7	17.3	18.0

(continued)

Table A.5 (*Continued*)

$p = 3$,
Upper Percentiles for $b_{1,p}$

	Percentiles					
n	90	92.5	95	97.5	99	99.9
60	1.5	1.6	1.8	2.0	2.4	2.9
70	1.3	1.4	1.5	1.7	2.0	2.5
80	1.13	1.2	1.3	1.5	1.7	2.2
90	1.01	1.08	1.16	1.3	1.5	1.9
100	.92	.97	1.05	1.18	1.3	1.7
150	.62	.66	.71	.80	.90	1.15
200	.47	.50	.54	.60	.68	.87
300	.32	.33	.36	.40	.46	.58
400	.237	.252	.272	.30	.34	.44
600	.159	.168	.182	.203	.230	.294
800	.119	.127	.137	.153	.173	.221
1000	.095	.010	.109	.122	.139	.177
1500	.064	.068	.073	.082	.093	.118
2000	.048	.051	.055	.061	.069	.089
3000	.032	.034	.037	.041	.046	.059
4000	.024	.025	.027	.031	.035	.044
5000	.019	.020	.022	.025	.028	.035

$p = 3$,
Upper and Lower Percentiles for $b_{2,p}$

	Percentiles							
n	1	2.5	5	10	90	95	97.5	99
60	12.1	12.5	12.8	13.1	16.1	16.6	17.2	17.9
70	12.3	12.6	13.0	13.2	16.1	16.6	17.1	17.7
80	12.4	12.8	13.1	13.3	16.1	16.5	17.0	17.6
90	12.5	12.9	13.2	13.5	16.0	16.5	16.9	17.5
100	12.6	13.0	13.3	13.5	16.0	16.4	16.8	17.4
150	13.0	13.3	13.6	13.8	15.9	16.2	16.5	17.0
200	13.2	13.5	13.8	14.0	15.8	16.1	16.3	16.8
300	13.6	13.8	14.0	14.2	15.7	15.9	16.1	16.5
400	13.7	13.9	14.1	14.3	15.6	15.8	16.0	16.3
600	13.9	14.1	14.3	14.4	15.51	15.67	15.81	15.97
800	14.1	14.2	14.3	14.5	15.45	15.59	15.71	15.85
1000	14.17	14.30	14.41	14.53	15.41	15.53	15.64	15.77
1500	14.33	14.43	14.52	14.62	15.34	15.44	15.53	15.63
2000	14.42	14.51	14.58	14.67	15.30	15.39	15.46	15.55
3000	14.53	14.60	14.66	14.73	15.25	15.32	15.38	15.45
4000	14.59	14.65	14.71	14.77	15.21	15.28	15.33	15.39
5000	14.63	14.69	14.74	14.80	15.19	15.25	15.30	15.35

$p = 4$,
Upper Percentiles for $b_{1,p}$

	Percentiles					
n	90	92.5	95	97.5	99	99.9
10	11.1	11.6	12.2	13.3	15.3	17.9
12	10.1	10.6	11.2	12.2	13.9	16.2

$p = 4$,
Upper and Lower Percentiles for $b_{2,p}$

	Percentiles							
n	1	2.5	5	10	90	95	97.5	99
10	17.0	17.3	17.6	17.8	21.5	22.4	23.0	24.0
12	17.4	17.7	18.0	18.3	22.3	23.3	24.2	25.4

(continued)

Table A.5 (*Continued*)

$p = 4$, Upper Percentiles for $b_{1,p}$							$p = 4$, Upper and Lower Percentiles for $b_{2,p}$								
	Percentiles							Percentiles							
n	90	92.5	95	97.5	99	99.9	n	1	2.5	5	10	90	95	97.5	99
14	9.2	9.7	10.2	11.2	12.7	14.8	14	17.7	18.0	18.3	18.6	23.0	24.0	25.0	26.1
16	8.4	8.8	9.4	10.3	11.6	13.6	16	18.0	18.2	18.6	18.9	23.4	24.4	25.4	26.6
18	7.7	8.0	8.7	9.5	10.7	12.6	18	18.2	18.4	18.8	19.2	23.8	24.7	25.8	26.9
20	7.0	7.4	8.0	8.8	9.9	11.6	20	18.4	18.6	19.0	19.4	24.0	25.0	26.1	27.1
25	5.9	6.2	6.6	7.1	8.1	9.7	25	18.8	19.1	19.5	19.8	24.5	25.4	26.4	27.3
30	5.0	5.3	5.6	6.0	6.8	8.1	30	19.1	19.4	19.8	20.2	24.7	25.5	26.6	27.4
40	3.9	4.1	4.3	4.6	5.2	6.2	40	19.6	19.9	20.3	21.0	25.0	25.7	26.7	27.4
50	3.1	3.3	3.5	3.8	4.2	5.0	50	20.0	20.3	20.6	21.0	25.1	25.7	26.6	27.3
60	2.7	2.8	2.9	3.2	3.5	4.2	60	20.2	20.5	20.9	21.3	25.14	25.7	26.6	27.2
70	2.3	2.4	2.5	2.8	3.0	3.7	70	20.4	20.7	21.0	21.5	25.15	25.7	26.5	27.0
80	2.0	2.1	2.2	2.4	2.7	3.2	80	20.6	21.0	21.2	21.7	25.15	25.6	26.4	26.9
90	1.81	1.89	2.0	2.2	2.4	2.9	90	20.8	21.1	21.4	21.8	25.14	25.6	26.3	26.8
100	1.64	1.71	1.81	1.97	2.2	2.6	100	20.9	21.2	21.5	21.9	25.12	25.6	26.2	26.7
150	1.11	1.16	1.22	1.33	1.46	1.76	150	21.4	21.7	22.0	22.33	25.03	25.42	25.9	26.3
200	.84	.87	.92	1.00	1.10	1.33	200	21.7	22.0	22.2	22.57	24.95	25.29	25.6	26.0
300	.56	.59	.62	.67	.74	.89	300	22.1	22.33	22.57	22.85	24.83	25.11	25.3	25.7
400	.42	.44	.47	.51	.56	.67	400	22.3	22.56	22.77	23.02	24.75	24.99	25.20	25.46
600	.282	.295	.31	.34	.37	.45	600	22.63	22.83	23.01	23.21	24.63	24.83	25.01	25.21
800	.212	.222	.234	.255	.280	.34	800	22.82	22.99	23.15	23.32	24.56	24.74	24.89	25.06
1000	.170	.177	.188	.204	.224	.271	1000	22.94	23.10	23.24	23.40	24.51	24.67	24.80	24.96
1500	.113	.118	.125	.136	.150	.181	1500	23.14	23.27	23.38	23.51	24.42	24.55	24.66	24.79
2000	.085	.089	.094	.102	.112	.136	2000	23.26	23.37	23.47	23.58	24.37	24.48	24.58	24.69
3000	.057	.059	.063	.068	.075	.091	3000	23.40	23.49	23.57	23.66	24.31	24.40	24.48	24.57
4000	.043	.045	.047	.051	.056	.068	4000	23.48	23.56	23.63	23.71	24.27	24.35	24.42	24.50
5000	.034	.039	.038	.041	.045	.054	5000	23.54	23.61	23.67	23.74	24.24	24.31	24.37	24.45

Table A.6 Upper Percentiles for Test of Single Multivariate Normal Outlier

Upper percentage points for the test statistic

$$D^2_{(n)} = \max_{1 \le i \le n} (\mathbf{y}_i - \bar{\mathbf{y}})' \mathbf{S}^{-1} (\mathbf{y}_i - \bar{\mathbf{y}}).$$

This tests for a single outlier in a sample of size n from a multivariate normal distribution. Reject and conclude that the outlier is significant if $D^2_{(n)}$ exceeds the table value.

	$p = 2$		$p = 3$		$p = 4$		$p = 5$	
n	$\alpha = .05$	$\alpha = .01$	$\alpha = .05$	$\alpha = .01$	$\alpha = .05$	$\alpha = .01$	$\alpha = .05$	$\alpha = .01$
5	3.17	3.19						
6	4.00	4.11	4.14	4.16				
7	4.71	4.95	5.01	5.10	5.12	5.14		
8	5.32	5.70	5.77	5.97	6.01	6.09	6.11	6.12
9	5.85	6.37	6.43	6.76	6.80	6.97	7.01	7.08
10	6.32	6.97	7.01	7.47	7.50	7.79	7.82	7.98
12	7.10	8.00	7.99	8.70	8.67	9.20	9.19	9.57
14	7.74	8.84	8.78	9.71	9.61	10.37	10.29	10.90
16	8.27	9.54	9.44	10.56	10.39	11.36	11.20	12.02
18	8.73	10.15	10.00	11.28	11.06	12.20	11.96	12.98
20	9.13	10.67	10.49	11.91	11.63	12.93	12.62	13.81
25	9.94	11.73	11.48	13.18	12.78	14.40	13.94	15.47
30	10.58	12.54	12.24	14.14	13.67	15.51	14.95	16.73
35	11.10	13.20	12.85	14.92	14.37	16.40	15.75	17.73
40	11.53	13.74	13.36	15.56	14.96	17.13	16.41	18.55
45	11.90	14.20	13.80	16.10	15.46	17.74	16.97	19.24
50	12.23	14.60	14.18	16.56	15.89	18.27	17.45	19.83
100	14.22	16.95	16.45	19.26	18.43	21.30	20.26	23.17
200	15.99	18.94	18.42	21.47	20.59	23.72	22.59	25.82
500	18.12	21.22	20.75	23.95	23.06	26.37	25.21	28.62

Table A.7 Upper Percentage Points of Hotelling's T^2 Distribution

Degrees of Freedom, ν	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$	$p=9$	$p=10$
					$\alpha = .05$					
2	18.513									
3	10.128	57.000								
4	7.709	25.472	114.986							
5	6.608	17.361	46.383	192.468						
6	5.987	13.887	29.661	72.937	289.446					
7	5.591	12.001	22.720	44.718	105.157	405.920				
8	5.318	10.828	19.028	33.230	62.561	143.050	541.890			
9	5.117	10.033	16.766	27.202	45.453	83.202	186.622	697.356		
10	4.965	9.459	15.248	23.545	36.561	59.403	106.649	235.873	872.317	
11	4.844	9.026	14.163	21.108	31.205	47.123	75.088	132.903	290.806	1066.774
12	4.747	8.689	13.350	19.376	27.656	39.764	58.893	92.512	161.967	351.421
13	4.667	8.418	12.719	18.086	25.145	34.911	49.232	71.878	111.676	193.842
14	4.600	8.197	12.216	17.089	23.281	31.488	42.881	59.612	86.079	132.582
15	4.543	8.012	11.806	16.296	21.845	28.955	38.415	51.572	70.907	101.499
16	4.494	7.856	11.465	15.651	20.706	27.008	35.117	45.932	60.986	83.121
17	4.451	7.722	11.177	15.117	19.782	25.467	32.588	41.775	54.041	71.127
18	4.414	7.606	10.931	14.667	19.017	24.219	30.590	38.592	48.930	62.746
19	4.381	7.504	10.719	14.283	18.375	23.189	28.975	36.082	45.023	56.587
20	4.351	7.415	10.533	13.952	17.828	22.324	27.642	34.054	41.946	51.884
21	4.325	7.335	10.370	13.663	17.356	21.588	26.525	32.384	39.463	48.184
22	4.301	7.264	10.225	13.409	16.945	20.954	25.576	30.985	37.419	45.202
23	4.279	7.200	10.095	13.184	16.585	20.403	24.759	29.798	35.709	42.750
24	4.260	7.142	9.979	12.983	16.265	19.920	24.049	28.777	34.258	40.699
25	4.242	7.089	9.874	12.803	15.981	19.492	23.427	27.891	33.013	38.961
26	4.225	7.041	9.779	12.641	15.726	19.112	22.878	27.114	31.932	37.469

(continued)

Table A.7 (*Continued*)

Degrees of Freedom, ν	$p = 1$	$p = 2$	$p = 3$	$p = 4$	$p = 5$	$p = 6$	$p = 7$	$p = 8$	$p = 9$	$p = 10$
					$\alpha = .05$					
27	4.210	6.997	9.692	12.493	15.496	18.770	22.388	26.428	30.985	36.176
28	4.196	6.957	9.612	12.359	15.287	18.463	21.950	25.818	30.149	35.043
29	4.183	6.919	9.539	12.236	15.097	18.184	21.555	25.272	29.407	34.044
30	4.171	6.885	9.471	12.123	14.924	17.931	21.198	24.781	28.742	33.156
35	4.121	6.744	9.200	11.674	14.240	16.944	19.823	22.913	26.252	29.881
40	4.085	6.642	9.005	11.356	13.762	16.264	18.890	21.668	24.624	27.783
45	4.057	6.564	8.859	11.118	13.409	15.767	18.217	20.781	23.477	26.326
50	4.034	6.503	8.744	10.934	13.138	15.388	17.709	20.117	22.627	25.256
55	4.016	6.454	8.652	10.787	12.923	15.090	17.311	19.600	21.972	24.437
60	4.001	6.413	8.577	10.668	12.748	14.850	16.992	19.188	21.451	23.790
70	3.978	6.350	8.460	10.484	12.482	14.485	16.510	18.571	20.676	22.834
80	3.960	6.303	8.375	10.350	12.289	14.222	16.165	18.130	20.127	22.162
90	3.947	6.267	8.309	10.248	12.142	14.022	15.905	17.801	19.718	21.663
100	3.936	6.239	8.257	10.167	12.027	13.867	15.702	17.544	19.401	21.279
110	3.927	6.216	8.215	10.102	11.934	13.741	15.540	17.340	19.149	20.973
120	3.920	6.196	8.181	10.048	11.858	13.639	15.407	17.172	18.943	20.725
150	3.904	6.155	8.105	9.931	11.693	13.417	15.121	16.814	18.504	20.196
200	3.888	6.113	8.031	9.817	11.531	13.202	14.845	16.469	18.083	19.692
400	3.865	6.052	7.922	9.650	11.297	12.890	14.447	15.975	17.484	18.976
1000	3.851	6.015	7.857	9.552	11.160	12.710	14.217	15.692	17.141	18.570
∞	3.841	5.991	7.815	9.488	11.070	12.592	14.067	15.507	16.919	18.307

(continued)

Table A.7 (*Continued*)

Degrees of Freedom, ν	$p = 1$	$p = 2$	$p = 3$	$p = 4$	$p = 5$	$p = 6$	$p = 7$	$p = 8$	$p = 9$	$p = 10$
					$\alpha = 0.01$					
2	98.503									
3	34.116	297.000								
4	21.198	82.177	594.997							
5	16.258	45.000	147.283	992.494						
6	13.745	31.857	75.125	229.679	1489.489					
7	12.246	25.491	50.652	111.839	329.433	2085.984				
8	11.259	21.821	39.118	72.908	155.219	446.571	2781.978			
9	10.561	19.460	32.598	54.890	98.703	205.293	581.106	3577.472		
10	10.044	17.826	28.466	44.838	72.882	128.067	262.076	733.045	4472.464	
11	9.646	16.631	25.637	38.533	58.618	93.127	161.015	325.576	902.392	5466.956
12	9.330	15.722	23.588	34.251	49.739	73.969	115.640	197.555	395.797	1089.149
13	9.074	15.008	22.041	31.171	43.745	62.114	90.907	140.429	237.692	472.742
14	8.862	14.433	20.834	28.857	39.454	54.150	75.676	109.441	167.499	281.428
15	8.683	13.960	19.867	27.060	36.246	48.472	65.483	90.433	129.576	196.853
16	8.531	13.566	19.076	25.626	33.672	44.240	58.241	77.755	106.391	151.316
17	8.400	13.231	18.418	24.458	31.788	40.975	52.858	68.771	90.969	123.554
18	8.285	12.943	17.861	23.487	30.182	38.385	48.715	62.109	80.067	105.131
19	8.185	12.694	17.385	22.670	28.852	36.283	45.435	56.992	71.999	92.134
20	8.096	12.476	16.973	21.972	27.734	34.546	42.779	52.948	65.813	82.532
21	8.017	12.283	16.613	21.369	26.781	33.088	40.587	49.679	60.932	75.181
22	7.945	12.111	16.296	20.843	25.959	31.847	38.750	46.986	56.991	69.389
23	7.881	11.958	16.015	20.381	25.244	30.779	37.188	44.730	53.748	64.719
24	7.823	11.820	15.763	19.972	24.616	29.850	35.846	42.816	51.036	60.879
25	7.770	11.695	15.538	19.606	24.060	29.036	34.680	41.171	48.736	57.671
26	7.721	11.581	15.334	19.279	23.565	28.316	33.659	39.745	46.762	54.953

(continued)

Table A.7 (*Continued*)

Degrees of Freedom, ν	$p = 1$	$p = 2$	$p = 3$	$p = 4$	$p = 5$	$p = 6$	$p = 7$	$p = 8$	$p = 9$	$p = 10$
					$\alpha = 0.01$					
27	7.677	11.478	15.149	18.983	23.121	27.675	32.756	38.496	45.051	52.622
28	7.636	11.383	14.980	18.715	22.721	27.101	31.954	37.393	43.554	50.604
29	7.598	11.295	14.825	18.471	22.359	26.584	31.236	36.414	42.234	48.839
30	7.562	11.215	14.683	18.247	22.029	26.116	30.589	35.538	41.062	47.283
35	7.419	10.890	14.117	17.366	20.743	24.314	28.135	32.259	36.743	41.651
40	7.314	10.655	13.715	16.750	19.858	23.094	26.502	30.120	33.984	38.135
45	7.234	10.478	13.414	16.295	19.211	22.214	25.340	28.617	32.073	35.737
50	7.171	10.340	13.181	15.945	18.718	21.550	24.470	27.504	30.673	33.998
55	7.119	10.228	12.995	15.667	18.331	21.030	23.795	26.647	29.603	32.682
60	7.077	10.137	12.843	15.442	18.018	20.613	23.257	25.967	28.760	31.650
70	7.011	9.996	12.611	15.098	17.543	19.986	22.451	24.957	27.515	30.139
80	6.963	9.892	12.440	14.849	17.201	19.536	21.877	24.242	26.642	29.085
90	6.925	9.813	12.310	14.660	16.942	19.197	21.448	23.710	25.995	28.310
100	6.895	9.750	12.208	14.511	16.740	18.934	21.115	23.299	25.496	27.714
110	6.871	9.699	12.125	14.391	16.577	18.722	20.849	22.972	25.101	27.243
120	6.851	9.657	12.057	14.292	16.444	18.549	20.632	22.705	24.779	26.862
150	6.807	9.565	11.909	14.079	16.156	18.178	20.167	22.137	24.096	26.054
200	6.763	9.474	11.764	13.871	15.877	17.819	19.720	21.592	23.446	25.287
400	6.699	9.341	11.551	13.569	15.473	17.303	19.080	20.818	22.525	24.209
1000	6.660	9.262	11.426	13.392	15.239	17.006	18.743	20.376	22.003	23.600
∞	6.635	9.210	11.345	13.277	15.086	16.812	18.475	20.090	21.666	23.209

Note: p = number of variables.

Table A.8 Bonferroni t-Values, $t_{\alpha/2k,\nu}$, $\alpha = 0.05$

	k									
	1	2	3	4	5	6	7	8	9	10
	$100\alpha/k$									
ν	5.0000	2.5000	1.6667	1.2500	1.0000	.8333	.7143	.6250	.5556	.5000
2	4.3027	6.2053	7.6488	8.8602	9.9248	10.8859	11.7687	12.5897	13.3604	14.0890
3	3.1824	4.1765	4.8567	5.3919	5.8409	6.2315	6.5797	6.8952	7.1849	7.4533
4	2.7764	3.4954	3.9608	4.3147	4.6041	4.8510	5.0675	5.2611	5.4366	5.5976
5	2.5706	3.1634	3.5341	3.8100	4.0321	4.2193	4.3818	4.5257	4.6553	4.7733
6	2.4469	2.9687	3.2875	3.5212	3.7074	2.8630	3.9971	4.1152	4.2209	4.3168
7	2.3646	2.8412	3.1276	3.3353	3.4995	3.6358	3.7527	3.8552	3.9467	4.0293
8	2.3060	2.7515	3.0158	3.2060	3.3554	3.4789	3.5844	3.6766	3.7586	3.8325
9	2.2622	2.6850	2.9333	3.1109	3.2498	3.3642	3.4616	3.5465	3.6219	3.6897
10	2.2281	2.6338	2.8701	3.0382	3.1693	3.2768	3.3682	3.4477	3.5182	3.5814
11	2.2010	2.5931	2.8200	2.9809	3.1058	3.2081	3.2949	3.3702	3.4368	3.4966
12	2.1788	2.5600	2.7795	2.9345	3.0545	3.1527	3.2357	3.3078	3.3714	3.4284
13	2.1604	2.5326	2.7459	2.8961	3.0123	3.1070	3.1871	3.2565	3.3177	3.3725
14	2.1448	2.5096	2.7178	2.8640	2.9768	3.0688	3.1464	3.2135	3.2727	3.3257
15	2.1314	2.4899	2.6937	2.8366	2.9467	3.0363	3.1118	3.1771	3.2346	3.2860
16	2.1199	2.4729	2.6730	2.8131	2.9208	3.0083	3.0821	3.1458	3.2019	3.2520
17	2.1098	2.4581	2.6550	2.7925	2.8982	2.9840	3.0563	3.1186	3.1735	3.2224
18	2.1009	2.4450	2.6391	2.7745	2.8784	2.9627	3.0336	3.0948	3.1486	3.1966
19	2.0930	2.4334	2.6251	2.7586	2.8609	2.9439	3.0136	3.0738	3.1266	3.1737
20	2.0860	2.4231	2.6126	2.7444	2.8453	2.9271	2.9958	3.0550	3.1070	3.1534
21	2.0796	2.4138	2.6013	2.7316	2.8314	2.9121	2.9799	3.0382	3.0895	3.1352
22	2.0739	2.4055	2.5912	2.7201	2.8188	2.8985	2.9655	3.0231	3.0737	3.1188
23	2.0687	2.3979	2.5820	2.7097	2.8073	2.8863	2.9525	3.0095	3.0595	3.1040
24	2.0639	2.3909	2.5736	2.7002	2.7969	2.8751	2.9406	2.9970	3.0465	3.0905
25	2.0595	2.3846	2.5660	2.6916	2.7874	2.8649	2.9298	2.9856	3.0346	3.0782
26	2.0555	2.3788	2.5589	2.6836	2.7787	2.8555	2.9199	2.9752	3.0237	3.0669
27	2.0518	2.3734	2.5525	2.6763	2.7707	2.8469	2.9107	2.9656	3.0137	3.0565
28	2.0484	2.3685	2.5465	2.6695	2.7633	2.8389	2.9023	2.9567	3.0045	3.0469
29	2.0452	2.3638	2.5409	2.6632	2.7564	2.8316	2.8945	2.9485	2.9959	3.0380
30	2.0423	2.3596	2.5357	2.6574	2.7500	2.8247	2.8872	2.9409	2.9880	3.0298
35	2.0301	2.3420	2.5145	2.6334	2.7238	2.7966	2.8575	2.9097	2.9554	2.9960
40	2.0211	2.3289	2.4989	2.6157	2.7045	2.7759	2.8355	2.8867	2.9314	2.9712
45	2.0141	2.3189	2.4868	2.6021	2.6896	2.7599	2.8187	2.8690	2.9130	2.9521
50	2.0086	2.3109	2.4772	2.5913	2.6778	2.7473	2.8053	2.8550	2.8984	2.9370
55	2.0040	2.3044	2.4694	2.5825	2.6682	2.7370	2.7944	2.8436	2.8866	2.9247
60	2.0003	2.2990	2.4630	2.5752	2.6603	2.7286	2.7855	2.8342	2.8768	2.9146
70	1.9944	2.2906	2.4529	2.5639	2.6479	2.7153	2.7715	2.8195	2.8615	2.8987
80	1.9901	2.2844	2.4454	2.5554	2.6387	2.7054	2.7610	2.8086	2.8502	2.8870
90	1.9867	2.2795	2.4395	2.5489	2.6316	2.6978	2.7530	2.8002	2.8414	2.8779
100	1.9840	2.2757	2.4349	2.5437	2.6259	2.6918	2.7466	2.7935	2.8344	2.8707
110	1.9818	2.2725	2.4311	2.5394	2.6213	2.6868	2.7414	2.7880	2.8287	2.8648
120	1.9799	2.2699	2.4280	2.5359	2.6174	2.6827	2.7370	2.7835	2.8240	2.8599
250	1.9695	2.2550	2.4102	2.5159	2.5956	2.6594	2.7124	2.7577	2.7972	2.8322
500	1.9647	2.2482	2.4021	2.5068	2.5857	2.6488	2.7012	2.7460	2.7850	2.8195
1000	1.9623	2.2448	2.3980	2.5022	2.5808	2.6435	2.6957	2.7402	2.7790	2.8133
∞	1.9600	2.2414	2.3940	2.4977	2.5758	2.6383	2.6901	2.7344	2.7729	2.8070

(continued)

Table A.8 (*Continued*)

	k								
	11	12	13	14	15	16	17	18	19
	$100\alpha/k$								
ν	.4545	.4167	.3846	.3571	.3333	.3125	.2941	.2778	.2632
2	14.7818	15.4435	16.0780	16.6883	17.2772	17.8466	18.3984	18.9341	19.4551
3	7.7041	7.9398	8.1625	8.3738	8.5752	8.7676	8.9521	9.1294	9.3001
4	5.7465	5.8853	6.0154	6.1380	6.2541	6.3643	6.4693	6.5697	6.6659
5	4.8819	4.9825	5.0764	5.1644	5.2474	5.3259	5.4005	5.4715	5.5393
6	4.4047	4.4858	4.5612	4.6317	4.6979	4.7604	4.8196	4.8759	4.9295
7	4.1048	4.1743	4.2388	4.2989	4.3553	4.4084	4.4586	4.5062	4.5514
8	3.8999	3.9618	4.0191	4.0724	4.1224	4.1693	4.2137	4.2556	4.2955
9	3.7513	3.8079	3.8602	3.9088	3.9542	3.9969	4.0371	4.0752	4.1114
10	3.6388	3.6915	3.7401	3.7852	3.8273	3.8669	3.9041	3.9394	3.9728
11	3.5508	3.6004	3.6462	3.6887	3.7283	3.7654	3.8004	3.8335	3.8648
12	3.4801	3.5274	3.5709	3.6112	3.6489	3.6842	3.7173	3.7487	3.7783
13	3.4221	3.4674	3.5091	3.5478	3.5838	3.6176	3.6493	3.6793	3.7076
14	3.3736	3.4173	3.4576	3.4949	3.5296	3.5621	3.5926	3.6214	3.6487
15	3.3325	3.3749	3.4139	3.4501	3.4837	3.5151	3.5447	3.5725	3.5989
16	3.2973	3.3386	3.3765	3.4116	3.4443	3.4749	3.5036	3.5306	3.5562
17	3.2667	3.3070	3.3440	3.3783	3.4102	3.4400	3.4680	3.4944	3.5193
18	3.2399	3.2794	3.3156	3.3492	3.3804	3.4095	3.4369	3.4626	3.4870
19	3.2163	3.2550	3.2906	3.3235	3.3540	3.3826	3.4094	3.4347	3.4585
20	3.1952	3.2333	3.2683	3.3006	3.3306	3.3587	3.3850	3.4098	3.4332
21	3.1764	3.2139	3.2483	3.2802	3.3097	3.3373	3.3632	3.3876	3.4106
22	3.1595	3.1965	3.2304	3.2618	3.2909	3.3181	3.3436	3.3676	3.3903
23	3.1441	3.1807	3.2142	3.2451	3.2739	3.3007	3.3259	3.3495	3.3719
24	3.1302	3.1663	3.1994	3.2300	3.2584	3.2849	3.3097	3.3331	3.3552
25	3.1175	3.1532	3.1859	3.2162	3.2443	3.2705	3.2950	3.3181	3.3400
26	3.1058	3.1412	3.1736	3.2035	3.2313	3.2572	3.2815	3.3044	3.3260
27	3.0951	3.1301	3.1622	3.1919	3.2194	3.2451	3.2691	3.2918	3.3132
28	3.0852	3.1199	3.1517	3.1811	3.2084	3.2339	3.2577	3.2801	3.3013
29	3.0760	3.1105	3.1420	3.1712	3.1982	3.2235	3.2471	3.2694	3.2904
30	3.0675	3.1017	3.1330	3.1620	3.1888	3.2138	3.2373	3.2594	3.2802
35	3.0326	3.0658	3.0962	3.1242	3.1502	3.1744	3.1971	3.2185	3.2386
40	3.0069	3.0393	3.0690	3.0964	3.1218	3.1455	3.1676	3.1884	3.2081
45	2.9872	3.0191	3.0482	3.0751	3.1000	3.1232	3.1450	3.1654	3.1846
50	2.9716	3.0030	3.0318	3.0582	3.0828	3.1057	3.1271	3.1472	3.1661
55	2.9589	2.9900	3.0184	3.0446	3.0688	3.0914	3.1125	3.1324	3.1511
60	2.9485	2.9792	3.0074	3.0333	3.0573	3.0796	3.1005	3.1202	3.1387
70	2.9321	2.9624	2.9901	3.0156	3.0393	3.0613	3.0818	3.1012	3.1194
80	2.9200	2.9500	2.9773	3.0026	3.0259	3.0476	3.0679	3.0870	3.1050
90	2.9106	2.9403	2.9675	2.9924	3.0156	3.0371	3.0572	3.0761	3.0939
100	2.9032	2.9327	2.9596	2.9844	3.0073	3.0287	3.0487	3.0674	3.0851
110	2.8971	2.9264	2.9532	2.9778	3.0007	3.0219	3.0417	3.0604	3.0779
120	2.8921	2.9212	2.9479	2.9724	2.9951	3.0162	3.0360	3.0545	3.0720
250	2.8635	2.8919	2.9178	2.9416	2.9637	2.9842	3.0034	3.0213	3.0383
500	2.8505	2.8785	2.9041	2.9276	2.9494	2.9696	2.9885	3.0063	3.0230
1000	2.8440	2.8719	2.8973	2.9207	2.9423	2.9624	2.9812	2.9988	3.0154
∞	2.8376	2.8653	2.8905	2.9137	2.9352	2.9552	2.9738	2.9913	3.0078

Table A.9 Lower Critical Values of Wilks' Λ, $\alpha = 0.05$

$$\Lambda = \frac{|\mathbf{E}|}{|\mathbf{E}+\mathbf{H}|} = \prod_{i=1}^{s} \frac{1}{1+\lambda_i},$$

where $\lambda_1, \lambda_2, \ldots, \lambda_s$ are eigenvalues of $\mathbf{E}^{-1}\mathbf{H}$. Reject H_0 if $\Lambda \leq$ table value.

	ν_H											
ν_E	1	2	3	4	5	6	7	8	9	10	11	12
						$p = 1$						
1	6.16[a]	2.50[a]	1.54[a]	1.11[a]	.868[a]	.712[a]	.603[a]	.523[a]	.462[a]	.413[a]	.374[a]	.341[a]
2	.098	.050	.034	.025	.020	.017	.015	.013	.011	.010	9.28[a]	8.51[a]
3	.229	.136	.097	.076	.062	.053	.046	.041	.036	.033	.030	.028
4	.342	.224	.168	.135	.113	.098	.086	.076	.069	.063	.058	.053
5	.431	.302	.236	.194	.165	.144	.128	.115	.104	.096	.088	.082
6	.501	.368	.296	.249	.215	.189	.169	.153	.140	.129	.119	.111
7	.556	.425	.349	.298	.261	.232	.209	.190	.175	.161	.150	.140
8	.601	.473	.396	.343	.303	.271	.246	.225	.208	.193	.180	.169
9	.638	.514	.437	.382	.341	.308	.281	.258	.239	.223	.209	.196
10	.668	.549	.473	.418	.376	.341	.313	.289	.269	.251	.236	.222
11	.694	.580	.505	.450	.407	.372	.343	.318	.297	.278	.262	.247
12	.717	.607	.534	.479	.436	.400	.370	.345	.323	.304	.286	.271
13	.736	.631	.560	.506	.462	.426	.396	.370	.347	.327	.310	.294
14	.753	.652	.583	.529	.486	.450	.420	.393	.370	.350	.332	.315
15	.768	.671	.603	.551	.508	.473	.442	.415	.392	.371	.352	.336
16	.781	.688	.622	.571	.529	.493	.462	.436	.412	.391	.372	.355
17	.792	.703	.639	.589	.548	.512	.482	.455	.431	.410	.390	.373
18	.803	.717	.655	.606	.565	.530	.499	.473	.449	.427	.408	.390
19	.813	.730	.669	.621	.581	.546	.516	.490	.466	.444	.425	.407
20	.821	.741	.683	.636	.596	.562	.532	.505	.482	.460	.440	.423
21	.829	.752	.695	.649	.610	.576	.547	.520	.497	.475	.455	.437
22	.836	.762	.706	.661	.623	.590	.561	.534	.511	.489	.470	.452
23	.843	.771	.717	.673	.635	.603	.574	.548	.524	.503	.483	.465
24	.849	.779	.727	.684	.647	.615	.586	.560	.537	.516	.496	.478
25	.855	.787	.736	.694	.658	.626	.598	.572	.549	.528	.508	.490
26	.860	.794	.744	.703	.668	.637	.609	.583	.560	.539	.520	.502
27	.865	.801	.752	.712	.677	.647	.619	.594	.571	.551	.531	.513
28	.870	.807	.760	.721	.686	.656	.629	.604	.582	.561	.542	.524
29	.874	.813	.767	.729	.695	.665	.638	.614	.592	.571	.552	.535
30	.878	.819	.774	.736	.703	.674	.647	.623	.601	.581	.562	.544
40	.907	.861	.824	.793	.766	.741	.718	.696	.677	.658	.641	.625
60	.938	.905	.879	.856	.835	.816	.798	.781	.766	.751	.736	.723
80	.953	.928	.907	.889	.873	.858	.843	.829	.816	.804	.792	.780
100	.962	.942	.925	.910	.897	.884	.872	.860	.849	.838	.828	.818
120	.968	.951	.937	.925	.913	.902	.891	.882	.872	.863	.854	.845
140	.973	.958	.946	.935	.925	.915	.906	.897	.889	.881	.873	.865
170	.978	.965	.955	.946	.937	.929	.922	.914	.907	.900	.893	.887
200	.981	.970	.962	.954	.947	.940	.933	.926	.920	.914	.908	.902
240	.984	.975	.968	.961	.955	.949	.944	.938	.933	.928	.923	.918
320	.988	.981	.976	.971	.966	.962	.957	.953	.949	.945	.941	.937
440	.991	.986	.982	.979	.975	.972	.969	.966	.963	.960	.957	.954
600	.994	.990	.987	.984	.982	.979	.977	.975	.972	.970	.968	.966
800	.995	.993	.990	.988	.986	.984	.983	.981	.979	.977	.976	.974
1000	.996	.994	.992	.991	.989	.988	.986	.985	.983	.982	.981	.979

[a]Multiply entry by 10^{-3}.

(continued)

Table A.9 (*Continued*)

	ν_H											
ν_E	1	2	3	4	5	6	7	8	9	10	11	12
						$p = 2$						
1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
2	2.50[a]	.641[a]	.287[a]	.162[a]	.104[a]	.072[a]	.053[a]	.041[a]	.032[a]	.026[a]	.022[a]	.018[a]
3	.050	.018	9.53[a]	5.84[a]	3.95[a]	2.85[a]	2.15[a]	1.68[a]	1.35[a]	1.11[a]	.928[a]	.787[a]
4	.136	.062	.036	.023	.017	.012	9.56[a]	7.62[a]	6.21[a]	5.17[a]	4.36[a]	3.73[a]
5	.224	.117	.074	.051	.037	.028	.023	.018	.015	.013	.011	.009
6	.302	.175	.116	.084	.063	.049	.040	.033	.027	.023	.020	.017
7	.368	.230	.160	.119	.092	.074	.060	.050	.042	.036	.032	.028
8	.4256	.280	.203	.155	.122	.099	.082	.069	.059	.051	.045	.040
9	.473	.326	.243	.190	.153	.126	.106	.090	.078	.068	.060	.053
10	.514	.367	.281	.223	.183	.152	.129	.111	.097	.085	.075	.067
11	.549	.404	.316	.255	.212	.179	.153	.133	.116	.102	.091	.082
12	.580	.437	.348	.286	.240	.204	.176	.154	.136	.120	.108	.097
13	.607	.467	.378	.314	.266	.229	.199	.175	.155	.138	.124	.112
14	.631	.495	.405	.340	.291	.252	.221	.195	.174	.156	.141	.128
15	.652	.519	.431	.365	.315	.275	.242	.215	.193	.174	.157	.143
16	.671	.542	.454	.389	.337	.296	.263	.235	.211	.191	.174	.159
17	.688	.562	.476	.410	.359	.317	.282	.254	.229	.208	.190	.174
18	.703	.581	.496	.431	.379	.337	.301	.272	.246	.225	.206	.189
19	.717	.598	.515	.450	.398	.355	.320	.289	.263	.241	.221	.204
20	.730	.614	.532	.468	.416	.373	.337	.306	.279	.256	.236	.218
21	.741	.629	.548	.485	.433	.390	.354	.322	.295	.271	.251	.232
22	.752	.643	.564	.501	.449	.406	.370	.338	.310	.286	.265	.246
23	.762	.656	.578	.516	.465	.422	.385	.353	.325	.300	.279	.259
24	.771	.668	.591	.530	.479	.436	.399	.367	.339	.314	.292	.272
25	.779	.679	.604	.544	.493	.450	.413	.381	.353	.328	.305	.285
26	.787	.689	.616	.556	.506	.464	.427	.395	.366	.341	.318	.297
27	.794	.699	.627	.568	.519	.477	.440	.407	.379	.353	.330	.309
28	.801	.708	.638	.580	.531	.489	.452	.420	.391	.365	.342	.321
29	.807	.717	.648	.591	.542	.501	.464	.432	.403	.377	.354	.332
30	.813	.725	.657	.601	.553	.512	.475	.443	.414	.388	.365	.344
40	.858	.786	.730	.682	.640	.602	.568	.537	.509	.484	.460	.439
60	.903	.853	.811	.774	.741	.710	.682	.656	.632	.609	.588	.568
80	.927	.888	.854	.825	.798	.772	.749	.727	.706	.686	.667	.649
100	.941	.909	.882	.857	.834	.813	.793	.774	.755	.738	.721	.705
120	.951	.924	.900	.879	.860	.841	.823	.807	.791	.775	.760	.746
140	.958	.934	.914	.895	.878	.862	.846	.831	.817	.803	.790	.777
170	.965	.946	.929	.913	.898	.885	.871	.859	.846	.834	.823	.812
200	.970	.954	.939	.926	.913	.901	.889	.878	.867	.857	.847	.837
240	.975	.961	.949	.938	.927	.917	.907	.897	.888	.879	.870	.862
320	.981	.971	.962	.953	.945	.937	.929	.922	.914	.907	.901	.894
440	.986	.979	.972	.965	.959	.953	.948	.942	.937	.932	.926	.921
600	.990	.984	.979	.975	.970	.966	.961	.957	.953	.949	.945	.942
800	.993	.988	.984	.981	.977	.974	.971	.968	.965	.962	.959	.956
1000	.994	.991	.987	.985	.982	.979	.977	.974	.972	.969	.967	.964

[a]Multiply entry by 10^{-3}.

(continued)

Table A.9 (*Continued*)

	ν_H											
ν_E	1	2	3	4	5	6	7	8	9	10	11	12
						$p = 3$						
1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
2	.000	.000	.000	.000	.000	.001[a]	.002[a]	.004[a]	.005[a]	.008[a]	.010[a]	.013[a]
3	1.70[a]	.354[a]	.179[a]	.127[a]	.105[a]	.095[a]	.091[a]	.090[a]	.091[a]	.092[a]	.095[a]	.098[a]
4	.034	.010	.004	.002	.001	.001	.809[a]	.659[a]	.562[a]	.496[a]	.449[a]	.416[a]
5	.097	.036	.018	.010	6.36[a]	4.37[a]	3.20[a]	2.46[a]	1.97[a]	1.64[a]	1.40[a]	1.22[a]
6	.168	.074	.040	.024	.016	.011	.008	.006	.004	3.94[a]	3.28[a]	2.79[a]
7	.236	.116	.068	.043	.029	.021	.016	.012	9.49[a]	7.67[a]	6.35[a]	5.35[a]
8	.296	.160	.099	.066	.046	.034	.026	.020	.016	.013	.011	9.00[a]
9	.349	.203	.131	.091	.066	.049	.038	.030	.024	.020	.016	.014
10	.396	.243	.164	.117	.086	.066	.052	.041	.034	.028	.023	.020
11	.437	.281	.196	.143	.108	.084	.067	.054	.044	.037	.031	.026
12	.473	.316	.226	.169	.130	.103	.083	.067	.056	.047	.040	.034
13	.505	.348	.255	.194	.152	.122	.099	.082	.068	.058	.049	.042
14	.534	.378	.283	.219	.174	.141	.116	.096	.081	.069	.059	.051
15	.560	.405	.309	.243	.195	.160	.133	.111	.095	.081	.070	.061
16	.583	.431	.334	.266	.216	.179	.149	.127	.108	.093	.081	.071
17	.603	.454	.357	.288	.236	.197	.166	.142	.122	.106	.092	.081
18	.622	.476	.379	.309	.256	.215	.183	.157	.136	.118	.104	.092
19	.639	.496	.399	.329	.275	.233	.199	.172	.149	.131	.115	.102
20	.655	.515	.419	.348	.293	.250	.215	.187	.163	.144	.127	.113
21	.669	.532	.437	.366	.310	.266	.230	.201	.177	.156	.139	.124
22	.683	.548	.454	.383	.327	.282	.246	.215	.190	.169	.150	.135
23	.695	.564	.470	.399	.343	.298	.260	.229	.203	.181	.162	.146
24	.706	.578	.486	.415	.359	.313	.275	.243	.216	.193	.173	.156
25	.717	.591	.500	.430	.374	.327	.289	.256	.229	.205	.185	.167
26	.727	.604	.514	.444	.388	.341	.302	.269	.241	.217	.196	.178
27	.736	.616	.527	.458	.401	.355	.315	.282	.253	.229	.207	.188
28	.744	.627	.540	.471	.415	.368	.328	.294	.265	.240	.218	.199
29	.752	.638	.552	.483	.427	.380	.340	.306	.277	.251	.229	.209
30	.760	.648	.563	.495	.439	.392	.352	.318	.288	.262	.239	.219
40	.816	.724	.651	.591	.539	.494	.454	.419	.387	.359	.334	.311
60	.875	.808	.752	.704	.661	.623	.587	.555	.526	.498	.473	.449
80	.905	.853	.808	.769	.733	.700	.670	.641	.615	.590	.566	.544
100	.924	.881	.844	.810	.780	.751	.725	.700	.676	.654	.632	.612
120	.936	.900	.868	.839	.813	.788	.764	.742	.721	.700	.681	.663
140	.945	.913	.886	.861	.837	.815	.794	.774	.755	.736	.719	.702
170	.955	.928	.905	.884	.864	.845	.827	.809	.792	.776	.761	.746
200	.961	.939	.919	.900	.883	.866	.850	.835	.820	.806	.792	.779
240	.968	.949	.932	.916	.901	.887	.873	.860	.848	.835	.823	.811
320	.976	.961	.948	.936	.925	.914	.903	.893	.883	.873	.864	.854
440	.982	.972	.962	.953	.945	.937	.929	.921	.913	.906	.899	.891
600	.987	.979	.972	.966	.959	.953	.947	.941	.936	.930	.924	.919
800	.990	.984	.979	.974	.969	.965	.960	.956	.951	.947	.943	.939
1000	.992	.987	.983	.979	.975	.972	.968	.964	.961	.957	.954	.950

[a]Multiply entry by 10^{-3}.

(continued)

Table A.9 (*Continued*)

	ν_H											
ν_E	1	2	3	4	5	6	7	8	9	10	11	12
						$p = 4$						
1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
2	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
3	.000	.000	.000	.000	.000	.001[a]	.001[a]	.001[a]	.002[a]	.002[a]	.002[a]	.003[a]
4	1.38[a]	.292[a]	.127[a]	.075[a]	.052[a]	.040[a]	.033[a]	.029[a]	.026[a]	.025[a]	.023[a]	.022[a]
5	.026	6.09[a]	2.31[a]	1.13[a]	.647[a]	.416[a]	.292[a]	.218[a]	.172[a]	.141[a]	.120[a]	.105[a]
6	.076	.024	.010	5.07[a]	2.90[a]	1.82[a]	1.22[a]	.872[a]	.652[a]	.508[a]	.409[a]	.338[a]
7	.135	.051	.024	.013	7.74[a]	4.94[a]	3.34[a]	2.36[a]	1.74[a]	1.33[a]	1.05[a]	.848[a]
8	.194	.084	.043	.025	.015	.010	6.98[a]	4.99[a]	3.70[a]	2.82[a]	2.21[a]	1.77[a]
9	.249	.119	.066	.040	.026	.017	.012	8.91[a]	6.66[a]	5.11[a]	4.01[a]	3.21[a]
10	.298	.155	.091	.057	.038	.027	.019	.014	.011	8.29[a]	6.54[a]	5.25[a]
11	.343	.190	.117	.077	.053	.037	.027	.021	.016	.012	9.84[a]	7.95[a]
12	.382	.223	.143	.097	.068	.049	.037	.028	.022	.017	.014	.011
13	.418	.255	.169	.117	.085	.063	.047	.037	.029	.023	.019	.015
14	.450	.286	.194	.138	.102	.077	.059	.046	.037	.030	.024	.020
15	.479	.314	.219	.159	.119	.091	.071	.056	.045	.037	.030	.025
16	.506	.340	.243	.180	.136	.106	.083	.067	.054	.044	.037	.031
17	.529	.365	.266	.200	.154	.121	.096	.078	.064	.053	.044	.037
18	.551	.389	.288	.219	.171	.136	.109	.089	.074	.061	.051	.044
19	.571	.410	.309	.239	.188	.151	.123	.101	.084	.070	.059	.051
20	.589	.431	.329	.257	.205	.166	.136	.113	.094	.079	.068	.058
21	.606	.450	.348	.275	.221	.181	.149	.124	.105	.089	.076	.065
22	.621	.468	.366	.292	.237	.195	.162	.136	.115	.098	.085	.073
23	.636	.485	.383	.309	.253	.210	.175	.148	.126	.108	.093	.081
24	.649	.501	.399	.325	.268	.224	.188	.160	.137	.118	.102	.089
25	.661	.516	.415	.340	.283	.237	.201	.172	.148	.128	.111	.097
26	.673	.530	.430	.355	.297	.251	.214	.183	.158	.138	.120	.106
27	.684	.544	.444	.369	.311	.264	.226	.195	.169	.147	.129	.114
28	.694	.556	.458	.383	.324	.277	.238	.206	.180	.157	.138	.122
29	.703	.568	.471	.396	.337	.289	.250	.217	.190	.167	.147	.131
30	.712	.580	.483	.409	.349	.301	.261	.228	.200	.177	.157	.139
40	.779	.668	.583	.513	.455	.406	.364	.327	.295	.267	.243	.221
60	.849	.767	.700	.643	.592	.547	.507	.471	.438	.409	.382	.357
80	.885	.821	.766	.718	.675	.636	.600	.567	.536	.508	.482	.457
100	.908	.854	.809	.768	.730	.696	.664	.634	.606	.580	.555	.532
120	.923	.877	.838	.802	.770	.739	.711	.684	.658	.634	.611	.590
140	.934	.894	.860	.828	.799	.772	.746	.721	.698	.676	.655	.635
170	.945	.912	.883	.856	.831	.808	.785	.764	.743	.724	.705	.687
200	.953	.925	.900	.876	.855	.834	.814	.795	.777	.759	.742	.726
240	.961	.937	.916	.896	.877	.859	.842	.826	.810	.795	.780	.765
320	.971	.952	.936	.921	.907	.893	.879	.866	.854	.841	.829	.818
440	.979	.965	.953	.942	.931	.921	.911	.901	.891	.882	.872	.863
600	.984	.974	.966	.957	.949	.941	.934	.926	.919	.912	.905	.898
800	.988	.981	.974	.968	.961	.956	.950	.944	.938	.933	.927	.922
1000	.991	.985	.979	.974	.969	.964	.960	.955	.950	.946	.941	.937

[a]Multiply entry by 10^{-3}.

(continued)

Table A.9 *(Continued)*

	ν_H											
ν_E	1	2	3	4	5	6	7	8	9	10	11	12
						$p = 5$						
1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
2	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
3	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
4	.000	.000	.000	.000	.001[a]	.001[a]	.001[a]	.001[a]	.001[a]	.001[a]	.001[a]	.001[a]
5	1.60[a]	.291[a]	.105[a]	.052[a]	.031[a]	.021[a]	.015[a]	.012[a]	.010[a]	.008[a]	.007[a]	.007[a]
6	.021	4.39[a]	1.48[a]	.647[a]	.335[a]	.197[a]	.126[a]	.087[a]	.064[a]	.049[a]	.039[a]	.032[a]
7	.063	.017	6.36[a]	2.90[a]	1.51[a]	.872[a]	.544[a]	.361[a]	.253[a]	.185[a]	.141[a]	.110[a]
8	.114	.037	.016	7.74[a]	4.21[a]	2.48[a]	1.56[a]	1.03[a]	.716[a]	.516[a]	.385[a]	.296[a]
9	.165	.063	.029	.015	8.79[a]	5.35[a]	3.43[a]	2.30[a]	1.61[a]	1.16[a]	.861[a]	.657[a]
10	.215	.092	.046	.026	.015	9.64[a]	6.34[a]	4.34[a]	3.06[a]	2.22[a]	1.66[a]	1.27[a]
11	.261	.122	.066	.038	.024	.015	.010	7.22[a]	5.17[a]	3.80[a]	2.86[a]	2.19[a]
12	.303	.153	.086	.053	.034	.022	.015	.011	7.99[a]	5.95[a]	4.51[a]	3.49[a]
13	.341	.183	.108	.068	.045	.031	.022	.016	.012	8.68[a]	6.66[a]	5.19[a]
14	.376	.212	.130	.085	.057	.040	.029	.021	.016	.012	9.31[a]	7.32[a]
15	.407	.239	.152	.102	.070	.050	.037	.027	.021	.016	.012	9.88[a]
16	.436	.266	.174	.119	.084	.061	.045	.034	.026	.020	.016	.013
17	.462	.291	.195	.136	.098	.072	.054	.042	.032	.025	.020	.016
18	.486	.315	.216	.154	.113	.084	.064	.050	.039	.031	.025	.020
19	.508	.337	.236	.171	.127	.096	.074	.058	.046	.037	.030	.024
20	.529	.359	.256	.188	.142	.109	.085	.067	.053	.043	.035	.029
21	.548	.379	.275	.205	.156	.121	.095	.076	.061	.050	.041	.034
22	.565	.398	.293	.221	.171	.134	.106	.085	.069	.057	.047	.039
23	.581	.416	.310	.237	.185	.146	.117	.095	.077	.064	.053	.044
24	.596	.433	.327	.253	.199	.159	.128	.104	.086	.071	.060	.050
25	.610	.449	.343	.268	.213	.171	.139	.114	.094	.079	.066	.056
26	.623	.465	.359	.283	.226	.183	.150	.124	.103	.087	.073	.062
27	.635	.479	.374	.297	.239	.195	.161	.134	.112	.094	.080	.068
28	.647	.493	.388	.311	.252	.207	.172	.143	.121	.102	.087	.075
29	.658	.506	.401	.324	.265	.219	.182	.153	.130	.110	.094	.081
30	.668	.519	.415	.337	.277	.230	.193	.163	.138	.118	.102	.088
40	.744	.617	.522	.446	.384	.333	.291	.255	.224	.198	.176	.156
60	.825	.729	.652	.587	.531	.482	.438	.400	.366	.336	.308	.284
80	.867	.791	.727	.672	.623	.578	.538	.502	.469	.438	.410	.385
100	.893	.830	.776	.728	.685	.645	.609	.576	.544	.516	.489	.464
120	.910	.856	.810	.768	.730	.694	.661	.631	.602	.575	.549	.525
140	.923	.876	.835	.798	.763	.731	.701	.673	.647	.621	.598	.575
170	.936	.897	.862	.830	.801	.773	.747	.722	.698	.675	.654	.633
200	.945	.912	.882	.854	.828	.803	.780	.758	.736	.716	.696	.677
240	.954	.926	.900	.877	.855	.833	.813	.793	.775	.757	.739	.722
300	.966	.944	.925	.906	.889	.872	.856	.841	.825	.811	.797	.783
440	.975	.959	.945	.931	.918	.905	.893	.881	.870	.858	.847	.836
600	.982	.970	.959	.949	.939	.930	.920	.911	.903	.894	.885	.877
800	.986	.977	.969	.961	.954	.947	.940	.933	.926	.919	.913	.906
1000	.989	.982	.975	.969	.963	.957	.951	.946	.940	.935	.929	.924

[a]Multiply entry by 10^{-3}.

(continued)

Table A.9 *(Continued)*

	ν_H											
ν_E	1	2	3	4	5	6	7	8	9	10	11	12
						$p = 6$						
1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
2	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
3	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
4	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
5	.007[a]	.002[a]	.001[a]	.001[a]	.001[a]	.000	.000	.000	.000	.000	.000	.000
6	2.04[a]	.315[a]	.095[a]	.040[a]	.021[a]	.012[a]	.008[a]	.006[a]	.004[a]	.003[a]	.003[a]	.002[a]
7	.019	3.48[a]	1.05[a]	.416[a]	.197[a]	.106[a]	.063[a]	.040[a]	.027[a]	.020[a]	.015[a]	.011[a]
8	.054	.013	4.37[a]	1.82[a]	.872[a]	.465[a]	.270[a]	.168[a]	.111[a]	.076[a]	.055[a]	.041[a]
9	.098	.029	.011	4.94[a]	2.48[a]	1.36[a]	.798[a]	.497[a]	.325[a]	.222[a]	.157[a]	.115[a]
10	.144	.050	.021	.010	5.35[a]	3.04[a]	1.83[a]	1.16[a]	.762[a]	.521[a]	.369[a]	.269[a]
11	.189	.074	.034	.017	9.64[a]	5.67[a]	3.51[a]	2.26[a]	1.51[a]	1.05[a]	.744[a]	.543[a]
12	.232	.099	.049	.027	.015	9.35[a]	5.94[a]	3.92[a]	2.66[a]	1.86[a]	1.34[a]	.983[a]
13	.271	.126	.066	.037	.022	.014	9.17[a]	6.17[a]	4.27[a]	3.03[a]	2.20[a]	1.63[a]
14	.308	.152	.084	.049	.031	.020	.013	9.07[a]	6.38[a]	4.59[a]	3.37[a]	2.52[a]
15	.341	.179	.103	.063	.040	.026	.018	.013	90[a]	6.57[a]	4.88[a]	3.68[a]
16	.372	.204	.122	.077	.050	.034	.024	.017	.012	8.97[a]	6.74[a]	5.14[a]
17	.400	.229	.141	.091	.061	.042	.030	.021	.016	.012	8.97[a]	6.90[a]
18	.426	.252	.160	.106	.072	.051	.037	.027	.020	.015	.012	8.97[a]
19	.450	.275	.179	.121	.084	.060	.044	.033	.025	.019	.015	.011
20	.473	.296	.197	.136	.096	.070	.052	.039	.030	.023	.018	.014
21	.493	.317	.215	.151	.109	.080	.060	.045	.035	.027	.021	.017
22	.512	.337	.233	.166	.121	.090	.068	.052	.041	.032	.025	.020
23	.530	.355	.250	.181	.134	.101	.077	.060	.047	.037	.030	.024
24	.546	.373	.266	.195	.146	.111	.086	.067	.053	.042	.034	.028
25	.562	.390	.282	.210	.159	.122	.095	.075	.060	.048	.039	.032
26	.576	.406	.298	.224	.171	.133	.104	.083	.066	.054	.044	.036
27	.590	.422	.313	.237	.183	.143	.113	.091	.073	.060	.049	.040
28	.603	.436	.327	.251	.195	.154	.123	.099	.080	.066	.054	.045
29	.615	.450	.341	.264	.207	.165	.132	.107	.088	.072	.060	.050
30	.626	.464	.355	.277	.219	.175	.142	.116	.095	.079	.066	.055
40	.711	.570	.467	.387	.324	.273	.232	.198	.170	.147	.127	.110
60	.802	.693	.608	.536	.476	.424	.379	.340	.305	.275	.249	.225
80	.849	.762	.690	.629	.574	.526	.483	.445	.410	.378	.350	.324
100	.878	.806	.745	.691	.642	.599	.559	.523	.489	.458	.430	.404
120	.898	.836	.783	.735	.692	.652	.616	.582	.551	.521	.494	.468
140	.912	.858	.811	.769	.730	.694	.660	.629	.599	.572	.546	.521
170	.927	.882	.842	.806	.772	.740	.710	.682	.656	.630	.607	.584
200	.938	.899	.864	.832	.803	.774	.748	.722	.698	.675	.653	.632
240	.948	.915	.886	.858	.833	.808	.785	.763	.741	.721	.701	.682
320	.961	.936	.913	.892	.872	.852	.834	.816	.799	.782	.766	.750
440	.972	.953	.936	.920	.905	.890	.876	.862	.849	.836	.823	.811
600	.979	.965	.953	.941	.930	.918	.908	.897	.887	.877	.867	.857
800	.984	.974	.964	.955	.947	.938	.930	.922	.914	.906	.898	.891
1000	.987	.979	.971	.964	.957	.950	.944	.937	.930	.924	.918	.912

[a]Multiply entry by 10^{-3}.

(continued)

Table A.9 *(Continued)*

	ν_H											
ν_E	1	2	3	4	5	6	7	8	9	10	11	12
						$p = 7$						
1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
2	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
3	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
4	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
5	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
6	.043[a]	.006[a]	.002[a]	.001[a]	.001[a]	.000	.000	.000	.000	.000	.000	.000
7	2.62[a]	.350[a]	.091[a]	.033[a]	.015[a]	.008[a]	.005[a]	.003[a]	.002[a]	.002[a]	.001[a]	.001[a]
8	.018	2.95[a]	.809[a]	.292[a]	.126[a]	.063[a]	.034[a]	.020[a]	.013[a]	.009[a]	.006[a]	.005[a]
9	.048	.010	3.20[a]	1.22[a]	.543[a]	.270[a]	.147[a]	.086[a]	.053[a]	.035[a]	.024[a]	.017[a]
10	.087	.023	8.07[a]	3.34[a]	1.56[a]	.798[a]	.440[a]	.259[a]	.160[a]	.104[a]	.070[a]	.049[a]
11	.128	.040	.016	6.97[a]	3.43[a]	1.83[a]	1.04[a]	.619[a]	.387[a]	.252[a]	.170[a]	.119[a]
12	.170	.060	.026	.012	6.34[a]	3.51[a]	2.05[a]	1.25[a]	.796[a]	.525[a]	.357[a]	.249[a]
13	.209	.083	.038	.019	.010	5.94[a]	3.57[a]	2.23[a]	1.45[a]	.967[a]	.665[a]	.468[a]
14	.246	.106	.052	.027	.015	9.17[a]	5.67[a]	3.63[a]	2.40[a]	1.62[a]	1.13[a]	.804[a]
15	.281	.129	.067	.037	.022	.013	8.37[a]	5.48[a]	3.68[a]	2.54[a]	1.79[a]	1.28[a]
16	.313	.153	.083	.047	.029	.018	.012	7.80[a]	5.34[a]	3.73[a]	2.66[a]	1.94[a]
17	.343	.176	.099	.059	.037	.024	.016	.011	7.38[a]	5.24[a]	3.78[a]	2.78[a]
18	.370	.199	.116	.071	.045	.030	.020	.014	9.81[a]	7.06[a]	5.16[a]	3.83[a]
19	.396	.221	.133	.083	.054	.037	.025	.018	.013	9.20[a]	6.80[a]	5.10[a]
20	.420	.242	.149	.096	.064	.044	.031	.022	.016	.012	8.72[a]	6.60[a]
21	.442	.263	.166	.109	.074	.052	.037	.026	.019	.014	.011	8.34[a]
22	.462	.283	.183	.123	.085	.060	.043	.031	.023	.018	.013	.010
23	.482	.301	.199	.136	.095	.068	.050	.037	.028	.021	.016	.013
24	.499	.320	.215	.149	.106	.077	.057	.042	.032	.025	.019	.015
25	.516	.337	.230	.162	.117	.086	.064	.048	.037	.029	.022	.018
26	.532	.354	.246	.175	.128	.095	.071	.055	.042	.033	.026	.020
27	.547	.370	.260	.188	.139	.104	.079	.061	.047	.037	.029	.024
28	.561	.385	.275	.201	.150	.113	.087	.068	.053	.042	.033	.027
29	.574	.399	.289	.214	.161	.123	.095	.074	.059	.047	.037	.030
30	.586	.413	.302	.226	.172	.132	.103	.081	.064	.052	.042	.034
40	.679	.526	.417	.335	.273	.224	.185	.154	.128	.108	.091	.077
60	.779	.660	.566	.490	.426	.373	.327	.288	.254	.225	.200	.178
80	.832	.735	.656	.588	.530	.479	.434	.394	.358	.326	.298	.272
100	.864	.783	.715	.656	.603	.556	.513	.475	.439	.408	.378	.352
120	.886	.817	.757	.704	.657	.613	.574	.537	.504	.473	.444	.418
140	.902	.841	.788	.741	.698	.658	.621	.587	.556	.526	.498	.472
170	.919	.868	.823	.782	.744	.709	.676	.645	.616	.589	.563	.539
200	.931	.887	.848	.812	.778	.747	.717	.689	.662	.637	.613	.590
240	.942	.905	.871	.841	.812	.784	.758	.733	.709	.687	.665	.644
320	.957	.928	.902	.878	.855	.833	.812	.792	.773	.754	.736	.719
440	.968	.947	.928	.910	.893	.876	.860	.844	.829	.814	.800	.786
600	.977	.961	.947	.933	.920	.908	.895	.883	.872	.860	.849	.838
800	.982	.971	.960	.950	.940	.930	.920	.911	.902	.893	.884	.876
1000	.986	.977	.968	.959	.951	.943	.936	.928	.921	.914	.906	.899

[a]Multiply entry by 10^{-3}.

(continued)

Table A.9 (*Continued*)

	ν_H											
ν_E	1	2	3	4	5	6	7	8	9	10	11	12
						$p = 8$						
1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
2	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
3	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
4	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
5	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
6	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
7	$.138^a$	$.015^a$	$.004^a$	$.001^a$	$.001^a$	.000	.000	.000	.000	.000	.000	.000
8	3.30^a	$.393^a$	$.090^a$	$.029^a$	$.012^a$	$.006^a$	$.003^a$	$.002^a$	$.001^a$	$.001^a$	$.001^a$	.000
9	.017	2.63^a	$.659^a$	$.218^a$	$.087^a$	$.040^a$	$.020^a$	$.011^a$	$.007^a$	$.004^a$	$.003^a$	$.002^a$
10	.044	8.63^a	2.46^a	$.872^a$	$.361^a$	$.168^a$	$.086^a$	$.047^a$	$.028^a$	4.017^a	$.011^a$	$.008^a$
11	.078	.019	6.15^a	2.36^a	1.03^a	$.497^a$	$.259^a$	$.144^a$	$.085^a$	$.052^a$	$.034^a$	$.023^a$
12	.116	.033	.012	4.99^a	2.30^a	1.16^a	$.619^a$	$.351^a$	$.209^a$	$.130^a$	$.084^a$	$.056^a$
13	.154	.051	.020	8.91^a	4.34^a	2.26^a	1.25^a	$.727^a$	$.441^a$	$.278^a$	$.181^a$	$.122^a$
14	.190	.070	.030	.014	7.22^a	3.92^a	2.23^a	1.33^a	$.824^a$	$.527^a$	$.347^a$	$.235^a$
15	.225	.090	.041	.021	.011	6.17^a	3.63^a	2.22^a	1.40^a	$.910^a$	$.608^a$	$.416^a$
16	.258	.111	.054	.028	.016	9.06^a	5.48^a	3.42^a	2.20^a	1.46^a	$.987^a$	$.683^a$
17	.289	.133	.067	.037	.021	.013	7.80^a	4.98^a	3.27^a	2.20^a	1.51^a	1.06^a
18	.318	.154	.082	.046	.027	.017	.011	6.92^a	4.62^a	3.15^a	2.19^a	1.56^a
19	.345	.175	.096	.056	.034	.021	.014	9.23^a	6.26^a	4.34^a	3.06^a	2.19^a
20	.370	.195	.111	.067	.042	.027	.018	.012	8.22^a	5.77^a	4.12^a	2.99^a
21	.393	.215	.127	.078	.050	.033	.022	.015	.010	7.46^a	5.39^a	3.95^a
22	.415	.235	.142	.089	.058	.039	.026	.018	.013	9.40^a	6.86^a	5.08^a
23	.436	.254	.157	.101	.067	.045	.031	.022	.016	.012	8.56^a	6.39^a
24	.455	.272	.172	.113	.076	.052	.037	.026	.019	.014	.010	7.88^a
25	.473	.289	.187	.124	.085	.060	.042	.031	.023	.017	.013	9.56^a
26	.490	.306	.201	.136	.095	.067	.048	.035	.026	.020	.015	.011
27	.505	.322	.215	.148	.104	.075	.055	.040	.030	.023	.017	.013
28	.520	.338	.229	.160	.114	.083	.061	.045	.034	.026	.020	.016
29	.534	.353	.243	.172	.124	.091	.068	.051	.039	.030	.023	.018
30	.548	.367	.256	.183	.134	.099	.074	.056	.043	.034	.026	.021
40	.649	.485	.372	.290	.229	.182	.146	.118	.096	.079	.065	.054
60	.758	.627	.527	.447	.381	.327	.282	.244	.212	.184	.161	.141
80	.815	.709	.623	.551	.489	.435	.389	.348	.313	.281	.253	.229
100	.851	.761	.687	.622	.566	.516	.471	.431	.395	.362	.333	.306
120	.875	.798	.732	.675	.623	.577	.535	.496	.461	.429	.399	.372
140	.892	.825	.767	.715	.667	.625	.585	.549	.515	.484	.455	.428
170	.911	.854	.804	.759	.717	.679	.644	.610	.579	.550	.523	.497
200	.924	.875	.831	.791	.755	.720	.688	.657	.629	.602	.576	.551
240	.936	.895	.858	.823	.791	.761	.732	.705	.679	.655	.631	.609
320	.952	.920	.891	.865	.839	.815	.792	.770	.748	.728	.708	.689
440	.965	.942	.920	.900	.880	.862	.844	.827	.810	.794	.778	.762
600	.974	.957	.941	.926	.911	.897	.883	.870	.857	.844	.831	.819
800	.981	.968	.955	.944	.933	.922	.911	.901	.890	.880	.871	.861
1000	.985	.974	.964	.955	.946	.937	.928	.920	.911	.903	.895	.887

[a] Multiply entry by 10^{-3}.

Table A.10 Upper Critical Values for Roy's Test, $\alpha = .05$

Roy's test statistic is given by

$$\theta = \frac{\lambda_1}{1 + \lambda_1},$$

where λ_1 is the largest eigenvalue of $\mathbf{E}^{-1}\mathbf{H}$. The parameters are

$$s = \min(\nu_H, p), \quad m = \frac{|\nu_H - p| - 1}{2}, \quad N = \frac{\nu_E - p - 1}{2}.$$

Reject H_0 if $\theta >$ table value.

	m								
N	0	1	2	3	4	5	7	10	15
					$s = 2$				
5	.565	.651	.706	.746	.776	.799	.834	.868	.901
10	.374	.455	.514	.561	.598	.629	.679	.732	.789
15	.278	.348	.402	.446	.483	.515	.567	.627	.696
20	.221	.281	.329	.369	.404	.434	.486	.546	.620
25	.184	.236	.278	.314	.346	.375	.424	.484	.558
30	.157	.203	.241	.274	.303	.330	.376	.433	.507
40	.122	.159	.190	.218	.243	.266	.306	.359	.428
50	.099	.130	.157	.180	.202	.222	.259	.306	.370
60	.084	.110	.133	.154	.173	.191	.223	.266	.326
80	.064	.085	.103	.119	.135	.149	.176	.211	.263
120	.043	.058	.070	.082	.093	.104	.123	.150	.190
240	.022	.030	.036	.042	.048	.054	.065	.080	.103
					$s = 3$				
5	.669	.729	.770	.800	.822	.840	.867	.894	.920
10	.472	.537	.586	.625	.656	.683	.725	.770	.819
15	.362	.422	.469	.508	.541	.569	.616	.669	.730
20	.293	.346	.390	.427	.458	.486	.533	.589	.656
25	.246	.294	.333	.367	.397	.424	.470	.525	.594
30	.212	.255	.291	.322	.350	.375	.419	.473	.543
40	.166	.201	.232	.259	.283	.305	.345	.395	.462
50	.136	.167	.192	.216	.237	.257	.292	.339	.402
60	.116	.142	.164	.185	.204	.221	.254	.296	.355
80	.089	.109	.127	.144	.160	.174	.201	.237	.288
120	.061	.075	.088	.100	.111	.122	.142	.169	.209
240	.031	.039	.046	.052	.058	.064	.075	.090	.114
					$s = 4$				
5	.739	.782	.813	.836	.854	.868	.889	.911	.933
10	.547	.601	.641	.674	.700	.723	.759	.798	.840
15	.431	.482	.523	.558	.587	.612	.654	.701	.756
20	.354	.402	.441	.474	.503	.529	.572	.623	.684
25	.301	.344	.380	.412	.440	.464	.507	.559	.624
30	.261	.301	.334	.364	.390	.414	.455	.507	.572
40	.207	.240	.269	.294	.318	.339	.377	.426	.490
50	.171	.199	.224	.247	.268	.287	.322	.367	.428
60	.145	.170	.193	.213	.232	.249	.280	.322	.380
80	.112	.132	.150	.167	.182	.196	.223	.259	.309
120	.077	.091	.104	.116	.127	.138	.158	.185	.226
240	.040	.047	.054	.061	.067	.073	.084	.100	.124

(continued)

Table A.10 (*Continued*)

	m								
N	0	1	2	3	4	5	7	10	15
					$s = 5$				
5	.788	.821	.845	.863	.877	.888	.906	.924	.942
10	.607	.651	.685	.713	.735	.755	.786	.820	.857
15	.488	.533	.569	.599	.625	.648	.685	.728	.777
20	.407	.449	.485	.515	.542	.565	.604	.651	.708
25	.349	.388	.422	.451	.477	.500	.540	.588	.648
30	.305	.341	.373	.400	.425	.448	.487	.535	.597
40	.243	.275	.302	.327	.349	.370	.406	.453	.514
50	.202	.230	.254	.276	.296	.315	.348	.392	.451
60	.173	.197	.219	.238	.257	.274	.304	.345	.401
80	.134	.154	.171	.188	.203	.217	.243	.278	.329
120	.093	.107	.120	.132	.143	.154	.174	.201	.241
240	.048	.056	.063	.069	.076	.082	.093	.109	.134
					$s = 6$				
5	.825	.850	.869	.883	.895	.904	.918	.934	.949
10	.655	.692	.721	.744	.764	.781	.808	.838	.871
15	.537	.576	.608	.635	.658	.678	.711	.750	.795
20	.454	.491	.523	.551	.575	.596	.632	.676	.728
25	.392	.428	.458	.485	.509	.531	.568	.613	.669
30	.345	.378	.407	.433	.457	.478	.514	.560	.618
40	.278	.307	.333	.356	.378	.397	.432	.477	.536
50	.232	.258	.281	.302	.322	.340	.372	.414	.472
60	.200	.223	.243	.262	.280	.297	.327	.366	.421
80	.156	.174	.192	.208	.222	.236	.262	.297	.346
120	.108	.122	.134	.146	.157	.168	.188	.215	.255
240	.056	.064	.071	.078	.084	.090	.101	.118	.142
					$s = 7$				
5	.852	.872	.887	.899	.908	.917	.929	.941	.955
10	.695	.726	.750	.771	.788	.802	.826	.853	.882
15	.579	.613	.641	.665	.686	.704	.734	.769	.810
20	.494	.528	.557	.582	.604	.624	.657	.697	.745
25	.431	.463	.491	.516	.538	.558	.593	.635	.688
30	.381	.412	.439	.463	.485	.505	.540	.583	.638
40	.309	.337	.362	.384	.404	.423	.456	.499	.555
60	.224	.246	.266	.285	.302	.318	.347	.386	.439
80	.176	.194	.211	.226	.241	.255	.280	.314	.363
100	.145	.160	.175	.188	.200	.212	.235	.265	.310
200	.077	.085	.093	.101	.109	.116	.129	.148	.175
300	.052	.058	.064	.069	.074	.079	.089	.103	.125
500	.032	.036	.039	.042	.046	.049	.055	.064	.078
1000	.016	.018	.020	.022	.023	.025	.028	.033	.041

(continued)

Table A.10 *(Continued)*

	m								
N	0	1	2	3	4	5	7	10	15
				$s = 8$					
5	.874	.890	.902	.912	.920	.927	.937	.948	.959
10	.728	.754	.775	.793	.808	.821	.842	.865	.892
15	.615	.645	.670	.692	.710	.727	.754	.786	.824
20	.531	.561	.587	.610	.630	.648	.679	.716	.761
25	.466	.495	.521	.544	.565	.583	.616	.655	.705
30	.414	.443	.468	.491	.511	.530	.563	.603	.655
40	.339	.365	.388	.409	.428	.446	.478	.519	.573
60	.248	.269	.288	.306	.323	.338	.367	.404	.456
80	.195	.213	.229	.244	.259	.272	.297	.330	.378
100	.161	.176	.190	.203	.216	.228	.250	.279	.323
200	.086	.094	.103	.110	.118	.125	.138	.157	.185
300	.058	.065	.070	.076	.081	.086	.096	.109	.130
500	.036	.040	.043	.047	.050	.053	.059	.068	.081
1000	.018	.020	.022	.024	.025	.027	.030	.035	.042
				$s = 9$					
5	.891	.904	.914	.922	.929	.935	.944	.953	.963
10	.756	.778	.797	.812	.825	.837	.855	.876	.901
15	.647	.674	.696	.715	.732	.747	.771	.801	.835
20	.563	.591	.614	.635	.654	.670	.698	.733	.775
25	.497	.525	.549	.570	.589	.606	.636	.673	.720
30	.445	.471	.495	.516	.535	.552	.583	.622	.671
40	.366	.391	.413	.433	.451	.468	.499	.538	.590
60	.270	.291	.309	.326	.343	.358	.385	.421	.472
80	.214	.231	.247	.262	.276	.289	.313	.346	.392
100	.177	.192	.206	.219	.231	.242	.264	.293	.336
200	.095	.104	.112	.119	.127	.134	.147	.166	.194
300	.065	.071	.077	.082	.087	.092	.102	.115	.136
500	.040	.043	.047	.051	.054	.057	.063	.072	.086
1000	.020	.022	.024	.026	.028	.029	.032	.037	.044
				$s = 10$					
5	.905	.916	.924	.931	.937	.941	.949	.958	.967
10	.780	.799	.815	.829	.840	.851	.867	.886	.908
15	.675	.699	.719	.736	.751	.764	.787	.814	.846
20	.592	.617	.639	.658	.675	.690	.716	.748	.787
25	.526	.551	.573	.593	.611	.627	.655	.690	.734
30	.473	.497	.519	.539	.557	.573	.603	.639	.686
40	.392	.415	.436	.455	.473	.489	.518	.555	.605
60	.292	.311	.329	.346	.361	.376	.402	.438	.487
80	.232	.249	.264	.278	.292	.305	.329	.361	.406
100	.193	.207	.220	.233	.245	.256	.278	.306	.348
200	.104	.112	.120	.128	.135	.142	.156	.174	.202
300	.071	.077	.083	.088	.093	.098	.108	.122	.143
500	.044	.047	.051	.054	.058	.061	.067	.076	.090
1000	.022	.024	.026	.028	.030	.031	.034	.039	.047

Table A.11 Upper Critical Values of Pillai's Statistic $V^{(s)}$, $\alpha = .05$

$$V^{(s)} = \sum_{i=1}^{s} \frac{\lambda_i}{1+\lambda_i},$$

where $\lambda_1, \lambda_2, \ldots, \lambda_s$ are eigenvalues of $\mathbf{E}^{-1}\mathbf{H}$. Reject H_0 if $V^{(s)}$ exceeds table value. The parameters s, m, and N are defined in Table A.10.

	N													
m	0	1	2	3	4	5	6	7	8	9	10	15	20	25
							$s = 2$							
0	1.536	1.232	1.031	.890	.782	.698	.629	.573	.526	.485	.451	.333	.263	.218
1	1.706	1.452	1.258	1.109	.991	.896	.817	.751	.694	.646	.604	.455	.364	.304
2	1.784	1.573	1.397	1.254	1.137	1.039	.956	.886	.825	.772	.725	.556	.451	.379
3	1.829	1.649	1.492	1.358	1.245	1.149	1.065	.993	.930	.875	.825	.643	.526	.445
4	1.859	1.703	1.560	1.436	1.329	1.235	1.153	1.081	1.018	.961	.910	.719	.594	.506
5	1.880	1.742	1.613	1.497	1.395	1.305	1.226	1.155	1.091	1.034	.983	.786	.655	.561
6	1.895	1.772	1.654	1.546	1.450	1.364	1.286	1.217	1.154	1.098	1.046	.846	.710	.612
7	1.907	1.796	1.687	1.586	1.495	1.413	1.338	1.270	1.209	1.153	1.102	.901	.761	.658
8	1.917	1.815	1.714	1.620	1.534	1.455	1.383	1.317	1.257	1.202	1.151	.950	.808	.702
9	1.924	1.831	1.737	1.649	1.567	1.491	1.422	1.358	1.299	1.245	1.195	.995	.851	.743
10	1.931	1.844	1.757	1.673	1.595	1.523	1.456	1.394	1.337	1.284	1.235	1.036	.891	.781
15	1.951	1.888	1.822	1.758	1.695	1.636	1.580	1.527	1.477	1.430	1.386			
20	1.963	1.913	1.860	1.807	1.756	1.706	1.658	1.612	1.568	1.527	1.487			
25	1.969	1.929	1.885	1.840	1.796	1.753	1.711	1.671	1.632	1.595	1.559			

(continued)

Table A.11 (*Continued*)

	N													
m	0	1	2	3	4	5	6	7	8	9	10	15	20	25
							$s = 3$							
0	2.037	1.710	1.473	1.294	1.153	1.040	.947	.869	.803	.746	.697	.524	.420	.350
1	2.297	1.988	1.751	1.564	1.412	1.287	1.183	1.094	1.017	.950	.892	.682	.552	.453
2	2.447	2.168	1.943	1.759	1.606	1.477	1.367	1.273	1.190	1.117	1.053	.818	.668	.565
3	2.544	2.294	2.084	1.907	1.757	1.628	1.517	1.420	1.334	1.258	1.190	.937	.772	.656
4	2.612	2.386	2.191	2.023	1.878	1.752	1.641	1.543	1.456	1.378	1.308	1.042	.866	.740
5	2.662	2.457	2.276	2.117	1.978	1.854	1.745	1.648	1.561	1.482	1.411	1.137	.952	.818
6	2.701	2.514	2.345	2.194	2.061	1.941	1.835	1.739	1.652	1.573	1.502	1.222	1.030	.890
7	2.732	2.559	2.402	2.259	2.131	2.016	1.912	1.818	1.732	1.654	1.582	1.300	1.103	.957
8	2.757	2.597	2.449	2.314	2.192	2.081	1.979	1.887	1.803	1.726	1.655	1.371	1.170	1.020
9	2.777	2.629	2.490	2.362	2.244	2.137	2.039	1.949	1.866	1.790	1.720	1.436	1.23	
10	2.795	2.656	2.525	2.403	2.291	2.187	2.092	2.004	1.923	1.848	1.779	1.496	1.3	
15	2.853	2.748	2.646	2.549	2.457	2.370	2.288	2.211	21.39	2.071	2.007			
20	2.885	2.802	2.718	2.637	2.560	2.485	2.414	2.347	2.283	2.222	2.163			
25	2.906	2.836	2.766	2.697	2.630	2.565	2.503	2.443	2.385					
							$s = 4$							
0	2.549	2.194	1.926	1.717	1.548	1.410	1.294	1.196	1.112	1.038	.974	.744	.602	
1	2.852	2.510	2.241	2.023	1.844	1.693	1.566	1.456	1.360	1.277	1.203	.932	.761	
2	3.052	2.733	2.472	2.256	2.074	1.919	1.786	1.670	1.567	1.477	1.396	1.097	.903	
3	3.193	2.898	2.650	2.440	2.260	2.104	1.969	1.849	1.743	1.649	1.564	1.243	1.032	
4	3.298	3.025	2.791	2.589	2.413	2.259	2.123	2.002	1.895	1.798	1.710	1.375	1.149	
5	3.378	3.126	2.905	2.711	2.541	2.390	2.255	2.135	2.027	1.929	1.840	1.494		
6	3.442	3.208	2.999	2.814	2.649	2.502	2.370	2.251	2.143	2.044	1.955	1.602		
7	3.494	3.276	3.079	2.902	2.743	2.600	2.470	2.353	2.246	2.148	2.058	1.70		
8	3.537	3.333	3.146	2.977	2.824	2.685	2.559	2.444	2.338	2.241	2.151	1.8		
9	3.574	3.382	3.205	3.043	2.896	2.761	2.638	2.525	2.421	2.325	2.236			
10	3.605	3.424	3.256	3.101	2.959	2.829	2.708	2.598	2.496	2.401	2.313			
15	3.710	3.570	3.436	3.310	3.191	3.079	2.974	2.876	2.783	2.696	2.615			
20	3.771	3.657	3.546	3.440	3.338	3.241	3.149							

(continued)

Table A.11 (*Continued*)

	N													
m	0	1	2	3	4	5	6	7	8	9	10	15	20	25
							$s = 5$							
0	3.055	2.681	2.389	2.155	1.962	1.801	1.664	1.547	1.445	1.356	1.277			
1	3.390	3.025	2.731	2.488	2.285	2.122	1.964	1.835	1.722	1.622	1.533			
2	3.628	3.281	2.993	2.751	2.545	2.367	2.213	2.077	1.957	1.850	1.754			
3	3.805	3.478	3.201	2.964	2.759	2.580	2.423	2.284	2.160	2.048	1.948			
4	3.941	3.635	3.370	3.140	2.938	2.761	2.604	2.463	2.337	2.222	2.119			
5	4.050	3.762	3.510	3.288	3.091	2.916	2.760	2.619	2.492	2.377	2.271			
6	4.138	3.868	3.627	3.414	3.223	3.052	2.897	2.758	2.630	2.514	2.408			
7	4.212	3.957	3.728	3.522	3.337	3.170	3.018	2.880						
8	4.274	4.033	3.815	3.617	3.438	3.275	3.126							
9	4.327	4.099	3.890	3.700	3.527	3.369								
10	4.372	4.156	3.957	3.774	3.607	3.45								
							$s = 6$							
0	3.559	3.171	2.859	2.604	2.390	2.209	2.053	1.918	1.799	1.694	1.601			
1	3.917	3.535	3.221	2.958	2.734	2.542	2.375	2.229	2.099	1.984	1.881			
2	4.185	3.817	3.508	3.245	3.018	2.821	2.647	2.494	2.358	2.235	2.125			
3	4.391	4.041	3.741	3.482	3.256	3.057	2.881	2.724	2.583	2.456	2.341			
4	4.556	4.223	3.934	3.681	3.458	3.260	3.084	2.925	2.782	2.652	2.534			
5	4.690	4.375	4.097	3.851	3.633	3.438	3.262	3.103	2.959	2.827	2.706			
6	4.802	4.502	4.236	3.998	3.785									
7	4.896	4.611	4.356	4.126	3.919									
8	4.976	4.706	4.461	4.239										
9	5.045	4.788	4.553											
10	5.106	4.860	4.635											

Table A.12 Upper Critical Values for the Lawley–Hotelling Statistic, $\alpha = .05$

The test statistic is $\nu_E U^{(s)}/\nu_H$, where $U^{(s)}$ is the Lawley–Hotelling statistic. Reject H_0 if $\nu_E U^{(s)}/\nu_H >$ table value.

	ν_H												
ν_E	2	3	4	5	6	8	10	12	15	20	25	40	60
							$p = 2$						
2[a]	9.8591	10.659	11.098	11.373	11.562	11.952	11.804	12.052	12.153	12.254	12.316	12.409	12.461
3	58.428	58.915	59.161	59.308	59.407	59.531	59.606	59.655	59.705	59.755	59.785	59.830	59.855
4	23.999	23.312	22.918	22.663	22.484	22.250	22.104	22.003	21.901	21.797	21.733	21.636	21.582
5	15.639	14.864	14.422	14.135	13.934	13.670	13.504	13.391	13.275	13.156	13.083	12.972	12.909
6	12.175	11.411	10.975	10.691	10.491	10.228	10.063	9.9489	9.8320	9.7118	9.6381	9.5251	9.4610
7	10.334	9.5937	9.1694	8.8927	8.6975	8.4396	8.2765	8.16399	8.0480	7.9285	7.8549	7.7417	7.6773
8	9.2069	8.4881	8.0752	7.8054	7.6145	7.3614	7.2008	7.0896	6.9748	6.8560	6.7826	6.6694	6.6048
10	7.9095	7.2243	6.8294	6.5702	6.3860	6.1405	5.9837	5.8745	5.7612	5.6433	5.5701	5.4564	5.3910
12	7.1902	6.5284	6.1461	5.8942	5.7147	5.4744	5.3200	5.2122	5.0997	4.9820	4.9085	4.7938	4.7274
14	6.7350	6.0902	4.7168	5.4703	5.2941	5.0574	4.9048	4.7977	4.6856	4.5678	4.4939	4.3780	4.3105
16	6.4217	5.7895	5.4230	5.1804	5.0067	4.7727	4.6213	4.5147	4.4028	4.2846	4.2102	4.0930	4.0243
18	6.1932	5.5708	5.2095	4.9700	4.7982	4.5663	4.4157	4.3094	4.1976	4.0791	4.0042	3.8855	3.8158
20	6.0192	5.4046	5.0475	4.8105	4.6402	4.4099	4.2600	4.1539	4.0420	3.9231	3.8477	3.7278	3.6569
25	5.7244	5.1237	4.7741	2.5415	4.3740	4.1465	3.9977	3.8919	3.7798	3.6598	3.5832	3.4605	3.3868
30	5.5401	4.9487	4.6040	4.3743	4.2086	3.9829	3.8347	3.7291	3.6166	3.4957	3.4181	3.2926	3.2168
35	5.4140	4.8291	4.8880	4.2604	4.0959	3.8715	3.7237	3.6181	3.5054	3.3836	3.3051	3.1774	3.1000
40	5.3224	4.7424	4.4039	4.1778	4.0143	3.7908	3.6433	3.5377	3.4247	3.3022	3.2230	3.0933	3.0140
50	5.1981	4.6249	4.2900	4.0661	3.9039	3.6817	3.5346	3.4289	3.3154	3.1919	3.1115	2.9787	2.8965
60	5.1178	4.5490	4.2166	3.9941	3.8328	3.6114	3.4646	3.3588	3.2450	3.1206	3.0392	2.9041	2.8196
70	5.0616	4.4960	4.1653	3.9439	3.7831	3.5624	3.4157	3.3099	3.1957	3.0706	2.9886	2.8516	2.7652
80	5.0200	4.4569	4.1275	3.9068	3.7465	3.5262	3.3796	3.2737	3.1594	3.0338	2.9512	2.8126	2.7247
100	4.9628	4.4030	4.0754	3.8557	3.6961	3.4764	3.3300	3.2240	3.1093	2.9829	2.8994	2.7586	2.6683
200	4.8514	4.2982	3.9742	3.7567	3.5983	3.3798	3.2336	3.1275	3.0120	2.8838	2.7984	2.6520	2.5559
∞	4.7442	4.1973	3.8769	3.6614	3.5044	3.2870	3.1410	3.0346	2.9182	2.7879	2.7002	2.5470	2.4428

[a]Multiply each entry in this row by 100.

(continued)

Table A.12 *(Continued)*

	ν_H											
ν_E	3	4	5	6	8	10	12	15	20	25	40	60
						$p = 3$						
3[a]	25.930	26.996	27.665	28.125	28.712	29.073	29.316	29.561	29.809	29.959	30.19	30.31
4[a]	1.1880	1.1929	1.1959	1.1978	1.2003	1.2018	1.2028	1.2038	1.2048	1.2054	1.2063	1.2068
5	42.474	41.764	1.305	40.983	40.562	40.300	40.120	39.937	39.750	39.635	39.462	39.366
6	25.456	24.715	24.235	23.899	23.458	23.182	22.992	22.799	22.600	22.479	22.294	22.190
7	18.752	18.056	17.605	17.288	16.870	16.608	16.427	16.241	16.051	15.934	15.755	15.653
8	15.308	14.657	14.233	13.934	13.540	13.290	13.118	12.941	12.758	12.646	12.473	12.375
10	11.893	11.306	10.921	10.649	10.287	10.057	9.8974	9.7320	9.5603	9.4541	9.2897	9.1955
12	10.229	9.6825	9.3234	9.0680	8.7271	8.5088	8.3566	8.1982	8.0330	7.9301	7.7700	7.6777
14	9.2550	8.7356	8.3935	8.1495	7.8225	7.6122	7.4649	7.3110	7.1497	7.0488	6.8908	6.7991
16	8.6180	8.1183	7.7884	7.5526	7.2355	7.0307	6.8868	6.7360	6.5772	6.4774	6.3204	6.2287
18	8.1701	7.6851	7.3644	7.1347	6.8251	6.6244	6.4830	6.3343	6.1771	6.0780	5.9212	5.8292
20	7.8384	7.3649	7.0513	6.8263	6.5224	6.3249	6.1853	6.0383	5.8822	5.7834	5.6266	5.5341
25	7.2943	6.8407	6.5394	6.3227	6.0287	5.8365	5.7001	5.5555	5.4010	5.3025	5.1446	5.0503
30	6.9654	6.5245	6.2311	6.0196	5.7319	5.5431	5.4085	5.2654	5.1116	5.0129	4.8535	4.7575
35	6.7453	6.3132	6.0253	5.8175	5.5341	5.3476	5.2143	5.0720	4.9185	4.8195	4.6586	4.5608
40	6.5877	6.1621	5.8783	5.6732	5.3929	5.2081	5.0757	4.9340	4.7806	4.6813	4.5189	4.4195
50	6.3773	5.9606	5.6823	5.4809	5.2050	5.0224	4.8911	4.7502	4.5967	4.4968	4.3319	4.2297
60	6.2433	5.8324	5.5577	5.3587	5.0856	4.9044	4.7739	4.6334	4.4798	4.3793	4.2123	4.1078
70	6.1504	5.7436	5.4715	5.2742	5.0031	4.8229	4.6929	4.5526	4.3988	4.2979	4.1292	4.0227
80	6.0823	5.6786	5.4084	5.2122	4.9426	4.7632	4.6336	4.4935	4.3395	4.2381	4.0680	3.9600
100	5.9891	5.5896	5.3220	5.1276	4.8601	4.6817	4.5525	4.4126	4.2583	4.1563	3.9840	3.8734
200	5.8099	5.4186	5.1562	4.9653	4.7017	4.5252	4.3970	4.2574	4.1023	3.9988	3.8212	3.7042
∞	5.6397	5.2565	4.9992	4.8116	4.5519	4.3773	4.2499	4.1104	3.9541	3.8487	3.6642	3.5384

[a] Multiply each entry in this row by 100.

(continued)

Table A.12 *(Continued)*

	ν_H										
ν_E	4	5	6	8	10	12	15	20	25	40	60
						$p = 4$					
4[a]	49.964	51.204	52.054	53.142	53.808	54.258	54.71	55.17	55.46	—	—
5[a]	1.9964	2.0013	2.0046	2.0087	2.0112	2.0128	2.0145	2.0171	2.0171	2.019	—
6	65.715	64.999	64.497	63.841	63.432	63.151	62.866	62.573	62.396	62.13	—
7	37.343	36.629	36.129	35.474	35.064	34.782	34.495	34.200	34.019	33.75	—
8	26.516	25.868	25.413	24.814	24.437	24.178	23.912	23.639	23.471	23.214	23.072
10	17.875	17.326	16.938	16.424	16.098	15.872	15.640	15.399	15.250	15.021	14.891
12	14.338	13.848	13.500	13.037	12.741	12.535	12.321	12.099	11.961	11.747	11.624
14	12.455	12.002	11.680	11.248	10.972	10.778	10.577	10.366	10.234	10.029	9.9103
16	11.295	10.868	10.563	10.154	9.8904	9.7054	9.5119	9.3085	9.1810	8.9808	8.8644
18	10.512	10.104	9.8121	9.4190	9.1647	8.9857	8.7978	8.5996	8.4748	8.2778	8.1626
20	9.9500	9.5560	9.2736	8.8926	8.6453	8.4708	8.2871	8.0926	7.9696	7.7748	7.6601
25	9.0585	8.6884	8.4223	8.0616	7.8261	7.6590	7.4821	7.2933	7.1730	6.9805	6.8659
30	8.5377	8.1825	7.9265	7.5784	7.3502	7.1876	7.0147	6.8291	6.7101	6.5181	6.4026
35	8.1968	7.8517	7.6026	7.2631	7.0397	6.8801	6.7099	6.5262	6.4079	6.2156	6.0989
40	7.9566	7.6188	7.3746	7.0413	6.8214	6.6640	6.4955	6.3131	6.1952	6.0023	5.8844
50	7.6404	7.3125	7.0751	6.7501	6.5350	6.3804	6.2143	6.0334	5.9157	5.7214	5.6011
60	7.4417	7.1202	6.8872	6.5676	6.3555	6.2027	6.0381	5.8581	5.7403	5.5446	5.4222
70	7.3054	6.9884	6.7584	6.4426	6.2325	6.0809	5.9173	5.7378	5.6200	5.4230	5.2987
80	7.2061	6.8924	6.6646	6.3515	6.1430	5.9924	5.8294	5.6503	5.5323	5.3343	5.2084
100	7.0711	6.7619	6.5372	6.2279	6.0215	5.8721	5.7101	5.5313	5.4131	5.2133	5.0849
200	6.8143	6.5139	6.2952	5.9933	5.7910	5.6439	5.4836	5.3053	5.1863	4.9819	4.8471
∞	6.5741	6.2821	6.0692	5.7743	5.5758	5.4309	5.2721	5.0940	4.9737	4.7629	4.6190

[a] Multiply each entry in this row by 100.

(continued)

Table A.12 (*Continued*)

ν_E	ν_H 5	6	8	10	12	15	20	25	40	60
					$p = 5$					
5[a]	81.991	83.352	85.093	86.160	86.88	—	—	—	—	—
6[a]	3.0093	3.0142	3.0204	3.0241	3.0266	3.0291	3.032	—	—	—
7	93.762	93.042	92.102	91.515	91.113	90.705	90.29	90.04	—	—
8	51.339	50.646	49.739	49.170	48.780	48.382	47.973	47.723	47.35	—
10	27.667	27.115	26.387	25.927	25.610	25.284	24.947	24.740	24.422	—
12	20.169	19.701	19.079	18.683	18.409	18.124	17.830	17.647	17.365	17.20
14	16.643	16.224	15.666	15.309	15.059	14.800	14.530	14.361	14.100	13.95
16	14.624	14.239	13.722	13.389	13.157	12.914	12.659	12.499	12.250	12.105
18	13.326	12.963	12.476	12.161	11.939	11.708	11.463	11.310	11.068	10.928
20	12.424	12.078	11.612	11.310	11.097	10.874	10.637	10.488	10.252	10.113
25	11.046	10.728	10.297	10.016	9.8168	9.6061	9.3814	9.2386	9.0102	8.8745
30	10.270	9.9689	9.5592	9.2907	9.0995	8.8964	8.6785	8.5389	8.3141	8.1790
35	9.7739	9.4836	9.0879	8.8277	8.6419	8.4437	8.2301	8.0926	7.8693	7.7339
40	9.4292	9.1469	8.7613	8.5070	8.3250	8.1303	7.9195	7.7833	7.5607	7.4247
50	8.9825	8.7107	8.3385	8.0921	7.9150	7.7248	7.5177	7.3829	7.1605	7.0229
60	8.7057	8.4406	8.0769	7.8355	7.6615	7.4741	7.2692	7.1351	6.9124	6.7730
70	8.5174	8.2570	7.8991	7.6612	7.4894	7.3039	7.1004	6.9667	6.7434	6.6024
80	8.3811	8.1241	7.7705	7.5351	7.3648	7.1807	6.9782	6.8448	6.6208	6.4785
100	8.1969	7.9446	7.5969	7.3649	7.1968	7.0145	6.8133	6.6801	6.4550	6.3103
200	7.8505	7.6070	7.2706	7.0451	6.8811	6.7023	6.5032	6.3702	6.1416	5.9908
∞	7.5305	7.2955	6.9698	6.7505	6.5902	6.4144	6.2171	6.0838	5.8499	5.6899

[a]Multiply each entry in this row by 100.

(continued)

Table A.12 (*Continued*)

	ν_H								
ν_E	6	8	10	12	15	20	25	40	60
					$p = 6$				
10	45.722	44.677	44.019	43.567	43.103	42.626	42.334	42.136	41.993
12	28.959	28.121	27.590	27.223	26.843	26.451	26.209	26.044	25.925
14	22.321	21.600	21.141	20.821	20.489	20.144	19.929	19.783	19.677
16	18.858	18.210	17.795	17.505	17.202	16.886	16.688	16.553	16.455
18	16.755	16.157	15.772	15.501	15.218	14.921	14.735	14.607	14.513
20	15.351	14.788	14.424	14.168	13.899	13.615	13.436	13.313	13.223
25	13.293	12.786	12.456	12.222	11.975	11.711	11.544	11.428	11.343
30	12.180	11.705	11.395	11.173	10.939	10.687	10.526	10.414	10.331
35	11.484	11.031	10.733	10.520	10.293	10.049	9.8921	9.7820	9.7003
40	11.009	10.571	10.282	10.075	9.8535	9.6142	9.4596	9.3508	9.2699
50	10.402	9.9832	9.7060	9.5067	9.2927	9.0598	8.9082	8.8009	8.7207
60	10.031	9.6246	9.3547	9.1602	8.9507	8.7215	8.5717	8.4651	8.3851
70	9.7813	9.3830	9.1182	8.9269	8.7204	8.4938	8.3450	8.2388	8.1589
80	9.6014	9.2093	8.9480	8.7591	8.5548	8.3300	8.1819	8.0759	7.9959
100	9.3598	8.9760	8.7197	8.5340	8.3326	8.1102	7.9629	7.8572	7.7771
200	8.9099	8.5419	8.2950	8.1153	7.9193	7.7011	7.5552	7.4494	7.3685
∞	8.4997	8.1463	7.9082	7.7340	7.5430	7.3284	7.1832	7.0768	6.9945

Table A.13 Orthogonal Polynomial Contrasts

		Variable										
p	Polynomial	1	2	3	4	5	6	7	8	9	10	$\mathbf{c}_i'\mathbf{c}_i$
3	Linear	−1	0	1								2
	Quadratic	1	−2	1								6
4	Linear	−3	−1	1	3							20
	Quadratic	1	−1	−1	1							4
	Cubic	−1	3	−3	1							20
5	Linear	−2	−1	0	1	2						10
	Quadratic	2	−1	−2	−1	2						14
	Cubic	−1	2	0	−2	1						10
	Quartic	1	−4	6	−4	1						70
6	Linear	−5	−3	−1	1	3	5					70
	Quadratic	5	−1	−4	−4	−1	5					84
	Cubic	−5	7	4	−4	−7	5					180
	Quartic	1	−3	2	2	−3	1					28
	Quintic	−1	5	−10	10	−5	1					252
7	Linear	−3	−2	−1	0	1	2	3				28
	Quadratic	5	0	−3	−4	−3	0	5				84
	Cubic	−1	1	1	0	−1	−1	1				6
	Quartic	3	−7	1	6	1	−7	3				154
	Quintic	−1	4	−5	0	5	−4	1				84
	Sextic	1	−6	15	−20	15	−6	1				924
8	Linear	−7	−5	−3	−1	1	3	5	7			168
	Quadratic	7	1	−3	−5	−5	−3	1	7			168
	Cubic	−7	5	7	3	−3	−7	−5	7			264
	Quartic	7	−13	−3	9	9	−3	−13	7			616
	Quintic	−7	23	−17	−15	15	17	−23	7			2,184
	Sextic	1	−5	9	−5	−5	9	−5	1			264
	Septic	−1	7	−21	35	−35	21	−7	1			3,432
9	Linear	−4	−3	−2	−1	0	1	2	3	4		60
	Quadratic	28	7	−8	−17	−20	−17	−8	7	28		2,772
	Cubic	−14	7	13	9	0	−9	−13	−7	14		990
	Quartic	14	−21	−11	9	18	9	−11	−21	14		2,002
	Quintic	−4	11	−4	−9	0	9	4	−11	4		468
	Sextic	4	−17	22	1	−20	1	22	−17	4		1,980
	Septic	−1	6	−14	14	0	−14	14	−6	1		858
	Octic	1	−8	28	−56	70	−56	28	−8	1		12,870
10	Linear	−9	−7	−5	−3	−1	1	3	5	7	9	330
	Quadratic	6	2	−1	−3	−4	−4	−3	−1	2	6	132
	Cubic	−42	14	35	31	12	−12	−31	−35	−14	42	8,580
	Quartic	18	−22	−17	3	18	18	3	−17	−22	18	2,860
	Quintic	−6	14	−1	−11	−6	6	11	1	−14	6	780
	Sextic	3	−11	10	6	−8	−8	6	10	11	3	660
	Septic	−9	47	−86	92	56	−56	−42	86	−47	9	29,172
	Octic	1	−7	20	−28	14	14	−28	20	−7	1	2,860
	Novic	−1	9	−36	84	−126	126	−84	36	−9	1	48,620

Note: Entries are rows $\mathbf{c}'_i$ of the $(p-1) \times p$ matrix $\mathbf{C}$ illustrated in (6.90) in Section 6.10.1

Table A.14 Test for Equal Covariance Matrices, $\alpha = 0.05$

ν	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$	$k=8$	$k=9$	$k=10$
					$p=2$				
3	12.18	18.70	24.55	30.09	35.45	40.68	45.81	50.87	55.86
4	10.70	16.65	22.00	27.07	31.97	36.75	41.45	46.07	50.64
5	9.97	15.63	20.73	25.57	30.23	34.79	39.26	43.67	48.02
6	9.53	15.02	19.97	24.66	29.19	33.61	37.95	42.22	46.45
7	9.24	14.62	19.46	24.05	28.49	32.83	37.08	41.26	45.40
8	9.04	14.33	19.10	23.62	27.99	32.26	36.44	40.57	44.64
9	8.88	14.11	18.83	23.30	27.62	31.84	35.98	40.05	44.08
10	8.76	13.94	18.61	23.05	27.33	31.51	35.61	39.65	43.64
11	8.67	13.81	18.44	22.85	27.10	31.25	35.32	39.33	43.29
12	8.59	13.70	18.30	22.68	26.90	31.03	35.08	39.07	43.00
13	8.52	13.60	18.19	22.54	26.75	30.85	34.87	38.84	42.76
14	8.47	13.53	18.10	22.42	26.61	30.70	34.71	38.66	42.56
15	8.42	13.46	18.01	22.33	26.50	30.57	34.57	38.50	42.38
16	8.38	13.40	17.94	22.24	26.40	30.45	34.43	38.36	42.23
17	8.35	13.35	17.87	22.17	26.31	30.35	34.32	38.24	42.10
18	8.32	13.30	17.82	22.10	26.23	30.27	34.23	38.13	41.99
19	8.28	13.26	17.77	22.04	26.16	30.19	34.14	38.04	41.88
20	8.26	13.23	17.72	21.98	26.10	30.12	34.07	37.95	41.79
25	8.17	13.10	17.55	21.79	25.87	29.86	33.78	37.63	41.44
30	8.11	13.01	17.44	21.65	25.72	29.69	33.59	37.42	41.21
					$p=3$				
4	22.41	35.00	46.58	57.68	68.50	79.11	89.60	99.94	110.21
5	19.19	30.52	40.95	50.95	60.69	70.26	79.69	89.03	98.27
6	17.57	28.24	38.06	47.49	56.67	65.69	74.58	83.39	92.09
7	16.59	26.84	36.29	45.37	54.20	62.89	71.44	79.90	88.30
8	15.93	25.90	35.10	43.93	52.54	60.99	69.32	77.57	85.73
9	15.46	25.22	34.24	42.90	51.33	59.62	67.78	75.86	83.87
10	15.11	24.71	33.59	42.11	50.42	58.57	66.62	74.58	82.46
11	14.83	24.31	33.08	41.50	49.71	57.76	65.71	73.57	81.36
12	14.61	23.99	32.67	41.00	49.13	57.11	64.97	72.75	80.45
13	14.43	23.73	32.33	40.60	48.65	56.56	64.36	72.09	79.72
14	14.28	23.50	32.05	40.26	48.26	56.11	63.86	71.53	79.11
15	14.15	23.32	31.81	39.97	47.92	55.73	63.43	71.05	78.60
16	14.04	23.16	31.60	39.72	47.63	55.40	63.06	70.64	78.14
17	13.94	23.02	31.43	39.50	47.38	55.11	62.73	70.27	77.76
18	13.86	22.89	31.26	39.31	47.16	54.86	62.45	69.97	77.41
19	13.79	22.78	31.13	39.15	46.96	54.64	62.21	69.69	77.11
20	13.72	22.69	31.01	39.00	46.79	54.44	61.98	69.45	76.84
25	13.48	22.33	30.55	38.44	46.15	53.70	61.16	68.54	75.84
30	13.32	22.10	30.25	38.09	45.73	53.22	60.62	67.94	75.18

(continued)

Table A.14 *(Continued)*

ν	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$	$k=8$	$k=9$	$k=10$
					$p=4$				
5	35.39	56.10	75.36	93.97	112.17	130.11	147.81	165.39	182.80
6	30.06	48.62	65.90	82.60	98.93	115.03	130.94	146.69	162.34
7	27.31	44.69	60.89	76.56	91.88	106.98	121.90	136.71	151.39
8	25.61	42.24	57.77	72.77	87.46	101.94	116.23	130.43	144.50
9	24.45	40.57	55.62	70.17	84.42	98.46	112.32	126.08	139.74
10	23.62	39.34	54.04	68.26	82.19	95.90	109.46	122.91	136.24
11	22.98	38.41	52.84	66.81	80.48	93.95	107.27	120.46	133.57
12	22.48	37.67	51.90	65.66	79.14	92.41	105.54	118.55	131.45
13	22.08	37.08	51.13	64.73	78.04	91.15	104.12	116.98	129.74
14	21.75	36.59	50.50	63.95	77.13	90.12	102.97	115.69	128.32
15	21.47	36.17	49.97	63.30	76.37	89.26	101.99	114.59	127.14
16	21.24	35.82	49.51	62.76	75.73	88.51	101.14	113.67	126.10
17	21.03	35.52	49.12	62.28	75.16	87.87	100.42	112.87	125.22
18	20.86	35.26	48.78	61.86	74.68	87.31	99.80	112.17	124.46
19	20.70	35.02	48.47	61.50	74.25	86.82	99.25	111.56	123.79
20	20.56	34.82	48.21	61.17	73.87	86.38	98.75	111.02	123.18
25	20.06	34.06	47.23	59.98	72.47	84.78	96.95	109.01	120.99
30	19.74	33.59	46.61	59.21	71.58	83.74	95.79	107.71	119.57
					$p=5$				
6	51.11	81.99	110.92	138.98	166.54	193.71	220.66	247.37	273.88
7	43.40	71.06	97.03	122.22	146.95	171.34	195.49	219.47	243.30
8	39.29	65.15	89.45	113.03	136.18	159.04	181.65	204.14	226.48
9	36.71	61.39	84.62	107.17	129.30	151.17	172.80	194.27	215.64
10	34.93	58.78	81.25	103.06	124.48	145.64	166.56	187.37	208.02
11	33.62	56.85	78.75	100.02	120.92	141.54	161.98	182.24	202.37
12	32.62	55.37	76.83	97.68	118.15	138.38	158.38	178.23	198.03
13	31.83	54.19	75.30	95.82	115.96	135.86	155.54	175.10	194.51
14	31.19	53.23	74.05	94.29	114.16	133.80	153.21	172.49	191.68
15	30.66	52.44	73.01	93.02	112.66	132.07	151.29	170.36	189.38
16	30.22	51.76	72.14	91.94	111.41	130.61	149.66	166.53	187.32
17	29.83	51.19	71.39	91.03	110.34	129.38	148.25	166.99	185.61
18	29.51	50.69	70.74	90.23	109.39	128.29	147.03	165.65	184.10
19	29.22	50.26	70.17	89.54	108.57	127.36	145.97	164.45	182.81
20	28.97	49.88	69.67	88.93	107.85	126.52	145.02	163.38	181.65
25	28.05	48.48	67.86	86.70	105.21	123.51	141.62	159.60	177.49
30	27.48	47.61	66.71	85.29	103.56	121.60	139.47	157.22	174.87

Note: Table contains upper percentage points for

$$-2\ln M = \nu\left(k\ln|\mathbf{S}| - \sum_{i=1}^{k}\ln|\mathbf{S}_i|\right)$$

for k samples, each with ν degrees of freedom. Reject $H_0 : \boldsymbol{\Sigma}_1 = \boldsymbol{\Sigma}_2 = \cdots = \boldsymbol{\Sigma}_k$ if $-2\ln M >$ table value.

Table A.15 Test for Independence of p Variables

Upper percentage points for

$$u' = -\left(\nu - \frac{2p+5}{6}\right) \ln\left(\frac{|\mathbf{S}|}{s_{11}\cdots s_{pp}}\right) = -\left(\nu - \frac{2p+5}{6}\right) \ln|\mathbf{R}|,$$

where ν is the degrees of freedom of $\mathbf{S}$ or $\mathbf{R}$. Reject independence if u' is greater than table value. The χ^2_α values are shown for comparison, since u' is approximately χ^2 distributed with $f = \frac{1}{2}p(p-1)$ degrees of freedom.

n	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$	$p=9$	$p=10$
				$\alpha = 0.05$				
4	8.020							
5	7.834	15.22						
6	7.814	13.47	24.01					
7	7.811	13.03	20.44	34.30				
8	7.811	12.85	19.45	28.75	46.05			
9	7.811	12.76	19.02	27.11	38.41	59.25		
10	7.812	12.71	18.80	26.37	36.03	49.42	73.79	
11	7.812	12.68	18.67	25.96	34.91	46.22	61.76	89.92
12	7.813	12.66	18.58	25.71	34.28	44.67	57.68	75.45
13	7.813	12.65	18.52	25.55	33.89	43.78	55.65	70.43
14	7.813	12.64	18.48	25.44	33.63	43.21	54.46	67.87
15	7.813	12.63	18.45	25.36	33.44	42.82	53.69	66.34
16	7.814	12.62	18.43	25.30	33.31	42.55	53.15	65.33
17	7.814	12.62	18.41	25.25	33.20	42.34	52.77	64.63
18	7.814	12.62	18.40	25.21	33.12	42.19	52.48	64.12
19	7.814	12.61	18.38	25.19	33.06	42.06	52.26	63.73
20	7.814	12.61	18.37	25.16	33.01	41.97	52.08	63.43
$\chi^2_{0.05}$	7.815	12.59	18.31	25.00	32.67	41.34	51.00	61.66
				$\alpha = 0.01$				
4	11.79							
5	11.41	21.18						
6	11.36	18.27	32.16					
7	11.34	17.54	26.50	44.65				
8	11.34	17.24	24.95	36.09	58.61			
9	11.34	17.10	24.29	33.63	47.05	74.01		
10	11.34	17.01	23.95	32.54	43.59	59.36	90.87	
11	11.34	16.96	23.75	31.95	42.00	54.83	73.03	109.53
12	11.34	16.93	23.62	31.60	41.13	52.70	67.37	88.05
13	11.34	16.90	23.53	31.36	40.59	51.49	64.64	81.20
14	11.34	16.89	23.47	31.20	40.23	50.73	63.06	77.83
15	11.34	16.87	23.42	31.09	39.97	50.22	62.05	75.84
16	11.34	16.86	23.39	31.00	39.79	49.85	61.36	74.56
17	11.34	16.86	23.36	30.94	39.65	49.59	60.86	73.66
18	11.34	16.85	23.34	30.88	39.54	49.38	60.49	73.01
19	11.34	16.85	23.32	30.84	39.46	49.22	60.21	72.52
20	11.34	16.84	23.31	30.81	39.39	49.09	59.99	72.15
$\chi^2_{0.01}$	11.34	16.81	23.21	30.58	38.93	48.28	58.57	69.92