MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR
SECOND YEAR FIRST SEMESTER

SCHOOL OF SCIENCE
BACHELOR OF SCIENCE IN MATHEMATICS
&
BACHELOR OF SCIENCE IN APPLIED STATISTICS WITH COMPUTING

COURSE CODE: STA 2113-1
COURSE TITLE: OPERATION RESEARCH I

DATE: 1ST APRIL, 2022 **TIME: 1100-1300**

INSTRUCTIONS TO CANDIDATES

1. Answer **ALL** questions from section A and any **TWO** from section B.
2. Use of sketch diagrams where necessary and brief illustrations are encouraged.
3. Read the instructions on the answer booklet keenly and adhere to them.

SECTION A (30 MARKS)

Answer all questions

QUESTION ONE (30 MARKS)

(a) Give four cases which occur when an LP is solved [4Mks]

(b) Consider the LPP

$$\min \quad z = 3x_1 - 3x_2 + 7x_3$$
$$\textit{subject to } x_1 + x_2 + 3x_3 \le 40$$
$$x_1 + 9x_2 - 7x_3 \ge 50$$
$$5x_1 + 3x_2 = 20$$
$$|5x_2 + 8x_3| \le 100$$
$$x_1 \ge 0, x_2 \ge 0, x_3 (urs)$$

Write the canonical and the standard forms of the LP model [6Mks]

(c) Give at least three importance of convexity in optimization [3Mks]

(d) Use simplex method to solve the following LP model

$$\max \quad z = 3x_1 + 5x_2 + 4x_3$$
$$\textit{subject to } 2x_1 + 3x_2 \le 8$$
$$+2x_2 + 5x_3 \le 10$$
$$3x_1 + 2x_2 + 4x_3 \le 15$$
$$x_1, x_2, x_3 \ge 0$$

[6Mks]

(e) Using the two-phase technique, solve the following LP model

$$\min \quad z = 3x_1 + 2x_2$$
$$\textit{subject to } x_1 + x_2 \ge 2$$
$$x_1 + 3x_2 \le 3$$
$$x_1 - x_2 = 1$$
$$x_1, x_2 \ge 0$$

[6Mks]

(f) Convert the following unbalanced transportation problem to a balanced transportation problem

		Destination			Supply
		1	2	3	
Source	1	30	50	15	300
	2	35	70	20	200
	3	20	45	60	500
Demand		300	200	400	900/1000

[5Mks]

SECTION B (40 MARKS)

Answer any TWO Questions

QUESTION TWO (20 MARKS)

(a) Consider the LP model

$$\max \quad z = 5x_1 + 12x_2 + 4x_3$$
$$\textit{subject to} \quad x_1 + 2x_2 + x_3 \le 5$$
$$2x_1 - x_2 + 3x_3 = 2$$
$$x_1, x_2, x_3 \ge 0$$

(i) Write the problem in standard form [2Mks]

(ii) Write down the dual problem [2Mks]

(iii) Solve the primal [5Mks]

(iv) Solve the dual [5Mks]

(b) Find the optimal assignment for the following problem

	1	2	3	4
1	1	4	6	3
2	9	7	10	9
3	4	5	11	7
4	8	7	8	5

[6Mks]

QUESTION THREE (20 MARKS)

(a) Consider the following LP model

$$\max \quad z = 10x_1 + 15x_2 + 20x_3$$
$$\textit{subject to} \quad 2x_1 + 4x_2 + 6x_3 \le 24$$
$$3x_1 + 9x_2 + 6x_3 \le 30$$
$$x_1, x_2, x_3 \ge 0$$

(i) Find the range of the objective function c_1 of the variable x_1 such that the optimality is unaffected
[4Mks]

(i) Find the range of the objective function c_2 of the variable x_2 such that the optimality is unaffected
[4Mks]

(ii) Check whether the optimality is affected if the profit coefficients are changed from $(10,15,20)$ to $(7,14,15)$. If so, find the revised optimum solution [4Mks]

(b) Consider the following LP model

$$\max \quad z = 3x_1 + 2x_2 + 5x_3$$
$$\text{subject to} \quad x_1 + 3x_2 + 2x_3 \le 15$$
$$2x_2 - x_3 \ge 5$$
$$2x_1 + x_2 - 5x_3 = 10$$
$$x_1, x_2, x_3 \ge 0$$

(i) Find its BVs [3Mks]

(ii) Use the result in (i) above to find optimal solution to the dual LP model [5Mks]

QUESTION FOUR (20 MARKS)

(a) Consider the LP model

$$\max \quad z = x_1 + 5x_2 + 3x_3$$
$$\text{subject to} \quad x_1 + 2x_2 + x_3 = 3$$
$$2x_1 - x_2 = 4$$
$$x_1, x_2, x_3 \ge 0$$

(i) Find the dual of the model [3Mks]

(ii) Given that basic variables are x_1 and x_3 in the primal optimal solution, find the dual optimal solution (dual variables value and dual objective function value) without solving the dual model [8Mks]

(b) Consider the LP model

$$\max \quad z = 6x_1 + 8x_2$$
$$\text{subject to} \quad 5x_1 + 10x_2 \le 60$$
$$4x_1 + 4x_2 \le 40$$
$$x_1, x_2 \ge 0$$

(i) Check whether the addition of the constraint $7x_1 + 2x_2 \le 65$ affects the optimality. If it does, find the new optimum solution [3Mks]

(ii) Check whether the addition of the constraint $6x_1 + 3x_2 \le 48$ affects the optimality. If it does, find the new optimum solution [6Mks]

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