## REGULAR UNIVERSITY EXAMINATIONS 2018/2019 ACADEMIC YEAR FIRST YEAR SECOND SEMESTER

## SCHOOL OF SCIENCE AND INFORMATION SCIENCES BACHELOR OF SCIENCE \& EDUCATION

## COURSE CODE: STA 1204/1207/1208 COURSE TITLE: PROBABILITY \& STATISTICS II

## INSTRUCTIONS TO CANDIDATES

1. Answer Question ONE and any other Two questions.
2. Show all the workings clearly
3. Do not write on the question paper
4. All Examination Rules Apply.

## Question One (30 Marks)

a) Let X be a random variable with pdf given by $f(x)=\left\{\begin{array}{ll}k \sqrt{x} & 0 \leq x \leq 9 \\ 0 & \text { elsewhere }\end{array}\right.$. Find the
(i) value of a constant $k$
(2 Marks)
(ii) cumulative distribution function $F(x)$
(1 Mark)
(iii) $P(X \leq 4)$
(3 Marks)
(iv) variance of X
(3 Marks)
b) The pdf of a random variable X is given by $f(x)= \begin{cases}\frac{x}{15} & x=1,2,3,4,5 \\ 0 & \text { elsewhere }\end{cases}$

Find the
i) $\quad P(1 \leq X<4)$
(3 Marks)
ii) the third moment of $X$ about the origin
c) If the probability that a person will believe a rumor about the transgressions of a certain politician is 0.4 . Find the probability that
i) The fourth person to hear this rumour is the second person to believe it.
(3 Marks)
ii) Let X be the number of people who fail to believe the rumour before we get the second person who believes it. Find the mean and variance of X .
(2 Marks)
d) The probability of launching a missile successfully is 0.8 . Test launches are conducted until three successful launches are achieved. Find the probability that exactly six launches will be required
(4 Marks)
e) The mgf of a random variable X is given by $m(t)=e^{2 t+\frac{25}{2} t^{2}}$.
i) what is the pdf of $X$
ii) Find $P(X<0)$ and $P(X>2)$

## Question Two (20 Marks)

a) At a certain airport, $80 \%$ of the flights arrive on time. A sample of 10 flights is studied. Let X be the number of flights that arrive on time. Find
i) $\quad P(X=10)$
ii) $\quad P(X=8)$
iii) $\quad P(X \geq 8)$
(8 Marks)
b) If the number of bacterial colonies on a petri dish follows a poisson distribution with average number $1.5 \mathrm{per} \mathrm{cm}^{2}$, find the probability that
(i) In $1.5 \mathrm{~cm}^{2}$ there are no bacterial colonies
(2 Marks)
(ii) in $2 \mathrm{~cm}^{2}$ there will be no bacterial colonies
(3 Marks)
(iii) in $1.5 \mathrm{~cm}^{2}$ there will be less than 3 bacterial colonies
(5 Marks)
(iv) In $4 \mathrm{~cm}^{2}$ there are six bacterial colonies.
(2 Marks)

## Question Three ( 20 Marks)

a) The probability that a computer running on a certain operating system crashes on any given day is 0.2 . Let X be the number of days on which the computer is working before crashing for the first time. Find
i) $\quad P(X=5)$
ii) $\quad P(X>2)$
iii) the mean and variance of $X$
(6 Marks)
b) The random variable X has pdf given by $f(x)= \begin{cases}\frac{1}{32}(x+4) & -4 \leq x \leq 4 \\ 0 & \text { elsewhere }\end{cases}$
i) Find the mean and variance of $X$
(3 Marks)
ii) Find $q$ such that $P(X \leq q)=\frac{1}{4}$
c) There are 15 restaurant in a certain town, four of them have health violations. A health inspector chooses 6 restaurants at random to visit. What is the probability that
i) two of the restaurants with health code violations will be visited
ii) more than two of the restaurants with health violations will be visited
(3 Marks)

## Question Four (20 Marks)

a) If $X \sim N(4,9)$. Find
i) $\quad P(1<X<7)$
ii) $P(|X|<6)$
(6 Marks)
b) The standard deviation of a certain group of 800 high school students' grades was $10 \%$ and the mean grade was $80 \%$. Assuming that the distribution is normal,
i) Find the number of students who scored more than 70\% (3 Marks)
ii) How many grades were above $90 \%$
(3 Marks)
iii) What was the highest grade of the lowest 100 high school students
(3 Marks)
c) Let the random variable X denote the waiting time (in minutes) for the next train. Under the assumption that a man arrives at the train station at random, X is distributed uniformly on the interval $(0,15)$.
i) Find the probability that he waits for at least 10 minutes for the train.
(3 Marks)
ii) Find the mean and variance of the waiting time.
(2 Marks)
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