MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATION
2018/2019 ACADEMIC YEAR
SECOND YEAR SECOND SEMESTER EXAMINATIONS

FOR

THE DEGREE OF BACHELOR SCIENCE (MATHEMATICS), APPLIED STATISTICS WITH COMPUTING AND EDUCATION (SCIENCE, ARTS AND SPECIAL NEEDS)

COURSE CODE: MAT 2212
COURSE TITLE: REAL ANALYSIS I

DATE 18TH APRIL 2019 **TIME: 1100 – 1300HRS**

INSTRUCTIONS TO CANDIDATES

1. This paper contains **FOUR** (4) questions
2. Answer question **ONE (1)** and any other **TWO** (**2**) questions
3. Do not forget to write your Registration Number.

QUESTION 1 (30MARKS)

a) Define power set $P(X)$ of a set X and hence show that the power set $P(\mathbb{N})$ of $\mathbb{N}$ is uncountable **5marks**

b) Given that $A=\left\{\frac{1}{n}: n\in\mathbb{N}\right\}$. Determine $\sup A$, $\inf A$ and state whether the maximum and minimum of A exists. **4marks**

c) Show that if $x\neq 0$, then $x^2>0$ and hence deduce that $1>0$ **4marks**

d) Prove that for a subset A of $\mathbb{R}$ that is bounded below $\inf A$ is unique **4marks**

e) Prove that $\sqrt{2}$ is irrational. **5marks**

f) Using the ratio test determine whether the following series converge or diverge $\sum_{n=1}^{\infty}\frac{n^2}{2^n}$ **3 marks**

g) Define the function $\rho:\mathbb{R}^2\times\mathbb{R}^2\to\mathbb{R}$ by $\rho(x,y)=|x_1-y_1|+|x_2-y_2|$ where $x=(x_1,x_2), y=(y_1,y_2)$. Show that ρ is a metric on $\mathbb{R}^2$ **5marks**

QUESTION 2 (20MKS)

a) Let A and B be non-void subsets of $\mathbb{R}$ that are bounded above. Show that $\sup(A+B)=\sup(A)+\sup(B)$ **5marks**

b) Show that the empty set ϕ is a subset of any other set **3marks**

c) Show that every convergent sequence is Cauchy **5marks**

d) Define a continuous function and hence determine whether the function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=\begin{cases}\frac{1}{x} & \text{if } x\neq 0\\ 0 & \text{if } x=0\end{cases}$ is continuous at $x=0$ **3marks**

e) Show that every Cauchy sequence is bounded **4marks**

QUESTION 3 (20MKS)

f) Show that a point $p \in X$ is a limit point of $E \subseteq X$ iff there exists a sequence $(x_n)^{\infty}$ of distinct points of E with $x_n \neq p$ $(\forall\, n \in \mathbb{N})$ such that $\lim_{n\to\infty} x_n = p$ **10marks**

g) Show that if the sequences (x_n) and (y_n) are convergent and $x_n \leq y_n$ for all $n \in \mathbb{N}$, then $\lim_{x\to\infty} x_n \leq \lim_{x\to\infty} y_n$ **5marks**

h) If $f(x) = \begin{cases} \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ find $f'(x)$. **5marks**

QUESTION 4 (20MKS)

a) Test for convergence in the following series

i. $\sum_{n=1}^{\infty} 2^{-n}$ ii. $\sum_{n=1}^{\infty} (-1)^{n+1}$ iii. $\sum_{n=1}^{\infty} n^{-1}$ **9marks**

b) Classify the monotonic sequences below.

i. $x_n = n^3$

ii. $x_n = (-1)^{n+1}$

iii. $x_n = \frac{1}{n}$

iv. $x_n = 2 \quad \forall\, n \in \mathbb{N}$ **4marks**

c) Binary operation $*$ on the set of all real numbers **R** is defined by $x * y = |x - y|$. Show that $*$ is commutative but not associative **2marks**

d) Define the terms

i. A metric space **1mark**

ii. Neighbourhood **1mark**

iii. A convergent sequence **1mark**

iv. Monotonic sequences **1mark**

v. Uniformly continuous function **1mark**

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