MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2018/2019 ACADEMIC YEAR
FIRST YEAR SECOND SEMESTER

SCHOOL OF SCIENCE
BACHELOR OF SCIENCE

COURSE CODE: MAT 1206
COURSE TITLE: LINEAR ALGEBRA I

DATE: 26-4-2019 **TIME: 11:00-13:00HRS**

INSTRUCTIONS TO CANDIDATES

Answer Question **ONE** and any other **TWO** questions

*This paper consists of **THREE** printed pages. Please turn over.*

QUESTION ONE (30 MARKS)

a) Let $\underline{u} = (1, -3, 4)$ and $\underline{v} = (3, 4, 7)$, find;

i. The angle between $\underline{u}$ and $\underline{v}$. **(4 marks)**

ii. Projection of $\underline{u}$ on $\underline{v}$. **(2 marks)**

iii. A vector orthogonal to both $\underline{u}$ and $\underline{v}$. **(3 marks)**

b) Determine whether the vectors $\underline{u} = (1,1,1)$, $\underline{v} = (2,-1,3)$ and $\underline{w} = (1,-5,3)$ are linearly dependent or linearly independent. **(6 marks)**

c) Find the condition on *a*, *b* and *c* such that $\underline{w} = (a,b,c)$ belongs to a space spanned by the vectors $\underline{u} = (1,-3,2)$ and $\underline{v} = (2,-1,1)$. **(6 marks)**

d) Find the basis for the null space and the nullity of the solution space to the following homogenous system.

$$\begin{aligned} 2x + 4y - 5z + 3t &= 0 \\ 3x + 6y - 7z + 4t &= 0 \\ 5x + 10y - 11z + 6t &= 0 \end{aligned}$$

(9 marks)

QUESTION TWO (20 MARKS)

a) Find the area of parallelogram with vertices at $\bar{A}(1,2,3)$, $\vec{B}(-3,2,5)$ and $\vec{C}(3,2,4)$. **(4 marks)**

b) Define linear combination, hence express vector $\underline{z} = (1,-2,5)$ as a linear combination of the vectors $\underline{u} = (1,1,1)$, $\underline{v} = (1,2,3)$ and $\underline{w} = (2,-1,1)$. **(6 marks)**

c) Let the mapping $f : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $f(x, y) = (x, x + y)$, show that f is a linear mapping. **(6 marks)**

d) Determine the values of *t* for which the matrix $A = \begin{bmatrix} t-2 & 4 & 3 \\ 1 & t+1 & -2 \\ 0 & 0 & t-4 \end{bmatrix}$ is singular. **(4 marks)**

QUESTION THREE (20 MARKS)

a) Determine the value of $k > 0$ such that $\|\underline{u}\| = \sqrt{39}$, where $\underline{u} = (1, k, -2, 5)$. (**4 marks)**

b) Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & 0 \\ -2 & -1 & 3 \\ -1 & 4 & -2 \end{bmatrix}$, find the rank of the matrix *A*. **(4 marks)**

c) For the following matrices $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$, show that

the associative law for matrix multiplication holds. **(4 marks)**

d) Determine the values of *k* so that the following system in unknowns *x, y* and *z*

$$x + y - z = 1$$
$$2x + 3y + kz = 3$$
$$x + ky + 3z = 2$$

has;

- **i.** A unique solution?
- **ii.** No solution?
- **iii.** infinitely many? **(8 marks)**

QUESTION FOUR (20 MARKS)

a) Solve the following system of linear equations by Gauss Jordan elimination.

$$x_1 + 3x_2 - 2x_3 + 2x_5 = 0$$
$$2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 = -1$$
$$5x_3 + 10x_4 + 15x_6 = 5$$
$$2x_1 + 6x_2 + 8x_4 + 4x_5 + 18x_6 = 6$$

(9 marks)

b) Verify dimension theorem for the linear transformation $T : \mathrm{R}^3 \to \mathrm{R}^3$ defined by $T(\mathrm{x, y, z}) = (\mathrm{x} + 2\mathrm{y} - \mathrm{z}, \mathrm{y} + \mathrm{z}, \mathrm{x} + \mathrm{y} - 2\mathrm{z})$. **(11 marks)**