MAASAI MARA UNIVERSITY

MAIN EXAMINATION 2018/2019 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER EXAMINATIONS

FOR

THE DEGREE OF BACHELOR OF SCIENCE

MAT 3227: NUMERICAL ANALYSIS II

DATE: TIME:

DURATION:

INSTRUCTIONS TO CANDIDATES

1. This paper contains **FOUR** (4) questions
2. Answer question **ONE (1)** and any other **TWO** (**2**) questions
3. Do not forget to write your Registration Number.

QUESTION ONE (THIRTY MARKS)

. a) Use Gaussian elimination with pivoting to solve the equations

$$2x_1 + x_2 + 5x_3 = 11$$
$$x_1 - 2x_2 + 3x_3 = 9$$
$$4x_1 - x_2 + x_3 = 7$$

(5 mks)

b) Given $\frac{dy}{dx} = 2x - y$, $y(0) = 1$, use the modified Euler's method to find $y(1)$ in five steps. **(6mks)**

c) Find the Taylor's series solution of the differential equation $\frac{dy}{dx} = x - y^2, y(0) = 1$ up to the term in x^5 . **(6mks)**

d) The data below is known to obey the law of the form $y = a + bx$

x	1	2	3	4	5
y	14	27	40	55	68

Find the least square line. **(5mks)**

e) Find the LU decomposition of the matrix

$$\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

Hence solve the equations

$$2x_1 + x_2 + 4x_3 = 12$$
$$8x_1 - 3x_2 + 2x_3 = 20$$
$$4x_1 + 11x_2 - x_3 = 33$$

(8mks)

QUESTION TWO (TWENTY MARKS)

a) Use Gauss-Siedel iterative method with $x^{(0)} = (0,0,0)$ to solve the equations

$$10x_1 - 2x_2 - x_3 - x_4 = 3$$
$$-2x_1 + 10x_2 - x_3 - x_4 = 15$$
$$-x_1 - x_2 + 10x_3 - 2x_4 = 27$$
$$-x_1 - x_2 - 2x_3 + 10x_4 = -9$$

Performing computations to 4dp and giving the final solution to 2dp. **(10mks)**

b) Use the fourth order Runge-Kutta methods to solve the differential equation

$\frac{dy}{dx} = xy + y^2 \quad y(0) = 1$ for y(0.1), y(0.2), y(0.3) **(10mks)**

QUESTION THREE (TWENTY MARKS)

a). Using $\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$, use the power method to get the dorminant eigen value of the matrix

$$\begin{bmatrix} 1 & 5 & -8 \\ 5 & -2 & 5 \\ -8 & 5 & 1 \end{bmatrix}$$

to the nearest whole number and the corresponding eigen vector with componentswhole numbers . Verify that $\begin{bmatrix} 1 & 0 & -1 \end{bmatrix}^T$ is also an eigen vector and state the corresponding eigen value.
Using the fact that eigen vectors of symmetric matrix are mutually orthogonal find the third eigen vector and the corresponding eigen value.

(16mks)

c) Given the data

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

Fit a power function of the form $y = ax^b$, a and b are constants. **(4mks)**

QUESTION FOUR (TWENTY MARKS)

a) Given the differential equation

$$\frac{dy}{dx} = x^2(1+y) \qquad y(1) = 1$$

i) Obtain the Taylor series solution up to the term in x^4. Use the series to compute y(1.1), y(1.2) and y(1.3).

(12mks)

ii) Use the predictor-Corrector method

$$\bar{y}_{n+1} = y_{n-3} + \frac{4h}{3}(2f_{n-2} - f_{n-1} + 2f_n)$$

$$y_{n+1} = y_{n-1} + \frac{h}{3}(f_{n-1} + 4f_n + f_{n+1})$$

to evaluate y(1.4). **(8mks)**

END