



MAASAI MARA UNIVERSITY

**REGULAR UNIVERSITY EXAMINATIONS
2018/2019 ACADEMIC YEAR
FIRST YEAR SECOND SEMESTER**

**SCHOOL OF SCIENCE
BACHELOR OF SCIENCE**

COURSE CODE: MAT 1206

COURSE TITLE: LINEAR ALGEBRA I

**DATE: 26-4-2019
13:00HRS**

TIME: 11:00-

INSTRUCTIONS TO CANDIDATES

Answer Question **ONE** and any other **TWO** questions

This paper consists of **THREE** printed pages. Please turn over.

QUESTION ONE (30 MARKS)

- a) Let $\underline{u} = (1, -3, 4)$ and $\underline{v} = (3, 4, 7)$, find;
- The angle between \underline{u} and \underline{v} . **(4 marks)**
 - Projection of \underline{u} on \underline{v} . **(2 marks)**
 - A vector orthogonal to both \underline{u} and \underline{v} . **(3 marks)**
- b) Determine whether the vectors $\underline{u} = (1, 1, 1)$, $\underline{v} = (2, -1, 3)$ and $\underline{w} = (1, -5, 3)$ are linearly dependent or linearly independent. **(6 marks)**
- c) Find the condition on a , b and c such that $\underline{w} = (a, b, c)$ belongs to a space spanned by the vectors $\underline{u} = (1, -3, 2)$ and $\underline{v} = (2, -1, 1)$. **(6 marks)**
- d) Find the basis for the null space and the nullity of the solution space to the following homogenous system.

$$2x + 4y - 5z + 3t = 0$$

$$3x + 6y - 7z + 4t = 0$$

$$5x + 10y - 11z + 6t = 0$$

(9 marks)

QUESTION TWO (20 MARKS)

- a) Find the area of parallelogram with vertices at $\bar{A}(1, 2, 3)$, $\bar{B}(-3, 2, 5)$ and $\bar{C}(3, 2, 4)$. **(4 marks)**
- b) Define linear combination, hence express vector $\underline{z} = (1, -2, 5)$ as a linear combination of the vectors $\underline{u} = (1, 1, 1)$, $\underline{v} = (1, 2, 3)$ and $\underline{w} = (2, -1, 1)$. **(6 marks)**
- c) Let the mapping $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be defined by $f(x, y) = (x, x + y)$, show that f is a linear mapping. **(6 marks)**

$$A = \begin{pmatrix} -2 & 4 & 3 \\ t+1 & -2 & \\ 0 & t-4 & \end{pmatrix}$$

- d) Determine the values of t for which the matrix is singular. **(4 marks)**

QUESTION THREE (20 MARKS)

- a) Determine the value of $k > 0$ such that $\|\underline{u}\| = \sqrt{39}$, where $\underline{u} = (1, k, -2, 5)$. (4 marks)

$$A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & 1 & 0 \\ 2 & -1 & 3 \\ 1 & 4 & -2 \end{pmatrix}$$

- b) Let A , find the rank of the matrix A . (4 marks)

$$A = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}, B = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \text{ and } C = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

- c) For the following matrices show that the associative law for matrix multiplication holds. (4 marks)

- d) Determine the values of k so that the following system in unknowns x, y and z
- $$\begin{aligned} x + y - z &= 1 \\ 2x + 3y + kz &= 3 \\ x + ky + 3z &= 2 \end{aligned}$$

has;

- i. A unique solution?
 - ii. No solution?
 - iii. infinitely many?
- (8 marks)

QUESTION FOUR (20 MARKS)

- a) Solve the following system of linear equations by Gauss Jordan elimination.

$$x_1 + 3x_2 - 2x_3 + 2x_5 = 0$$

$$2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 = -1$$

$$5x_3 + 10x_4 + 15x_6 = 5$$

$$2x_1 + 6x_2 + 8x_4 + 4x_5 + 18x_6 = 6 \quad (9 \text{ marks})$$

- b) Verify dimension theorem for the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined

by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$. (11 marks)