MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2018/2019 ACADEMIC YEAR
SECOND YEAR TWO SEMESTER

SCHOOL OF SCIENCE
BSC. MATHEMATICS

COURSE CODE: MAT 2215
COURSE TITLE: GROUP THEORY 1

DATE: 25TH APRIL, 2019 **TIME: 0830 – 1030 HRS**

INSTRUCTIONS TO CANDIDATES

1. Answer Question **ONE** and any other **TWO** questions.
2. All Examination Rules Apply.

<u>QUESTION 1</u> [30 MARKS]

1 a). Give an example of:

i. an associative binary operation on a set.
ii. anon-associative binary operation.

In each case specify the operation and the set, and support your claim.

[5 Marks]

1 b). (i.) Give the meaning of the statement:

" G is a non –commutative group with two generators".

i. Show that S_3 a non-commutative group.
ii. Give an example of a group of order 4 with two generators.

[6 Marks]

1 c). State the ring axioms and give an example of a ring with a finite number of elements. [5 Marks]

1 d). i. Write down the elements of the field Z_5 and construct a multiplication table for the field.

ii. Solve the equation x^2 =4 over Z_5 . [6Marks]

1 e). Give the definition of:

i. Subgroup.
ii. Coset.
iii. Factor group.

Illustrate using a group of your choice. [8 Marks]

<u>QUESTION 2</u> [20 MARKS]

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ be matrices over $\mathbb{Q}$:

i. Determine A^2 , B^2 , AB , BA , $(AB)^{15}$, $(AB)^n$.
ii. List all elements of multiplicative group V generated by A and .
iii. If $\begin{bmatrix} 0 & b \\ c & 0 \end{bmatrix}^2 =$¿ B , determine possible integer values of b and c and the order of the group generated by A and $\begin{bmatrix} 0 & b \\ c & o \end{bmatrix}$.

<u>QUESTION 3</u> [20 MARKS]

3 a. State and prove Lagrange an finite groups.

3 b. Let $D =< a , b \mid a^2 = b^3 = e, ba = ab^2 >$

i. List the elements of D .
ii. List all the subgroups of D .
iii. Choose a subgroup of order 2, of D and use it to illustrate Lagrange`s theorem.

<u>QUESTION 4</u> [20 MARKS]

4 a. Let R be the ring < Z_{12}¿ ;+¿$_{12,}$ $\times_{12}$ >:

i. Write down all the non-zero divisors of zero in R .
ii. Determine all the elements with multiplicative inverses and show that they form a group.
iii. Determine the ideals of R .
iv. For each ideal in iii. Determine the corresponding factor ring.

4 b. Let C be the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ in the ring R of 2×2 matrices

Over $\mathbb{Z}$;

i. Write down 5 matrices D such that $CD = 0$ and $D \neq 0$.
ii. Determine whether the set of all such matrices D such that $CD = 0$ is a right ideal of the ring R.

<u>QUESTION 5</u> [20 MARKS]

5 a. Give an example of:

i. A finite field with n elements where $5 < n < 9$.
ii. A field with an infinite number of elements.
iii. A field with an uncountable number of elements.

5 b. Let $\{f(x) = x^2 + x + 1\}$ be a polynomial in Z_2 [x], and

let α be a root of f(x).

i. Show that {0, 1, α, α+1} is a field with 4 elements.
ii. Solve the equation $x^3 - 1 = 0$ in the field.

5 c. A function:

F: $Z_{12} \to Z_{12}$ is given by $f(x) = 4x$.

Determine the image of f and the set of elements that are mapped to zero.

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