MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2016/2017 ACADEMIC YEAR
YEAR II SEMESTER I

SCHOOL OF MATHEMATICAL AND PHYSICAL SCIENCES
BACHELOR OF SCIENCE

COURSE CODE:STA 1208
COURSE TITLE: PROBABILITY AND STATISTICS II

DATE: 4TH DEC, 2018 **TIME:11-1PM**

INSTRUCTIONS TO CANDIDATES

1. Answer Question **ONE** and any other **TWO** questions.
2. All Examination Rules Apply.

Question 1(30 Marks)

a) Define the following terms as used in statistics..

i) Discrete random variable (2mks)

ii) Continues random variable (2mks)

iii) Probability mass function (2mks)

iv) Probability density function (2mks)

b) The random variable Y has a Poisson distribution with pmf.

$$f(y) = \frac{e^{-\lambda}\lambda^{y}}{y!}, y > 0; \lambda > 0$$

(i) Show that the mgf of Y is $M_Y(t) = e^{\lambda(e^t - 1)}$ (3mks)

(ii) Use mgf to show that $E(Y) = \lambda$.Given also that $E(Y^2) = \lambda + \lambda^2$ and $E(Y^2) = \lambda + 3\lambda^2 + \lambda^3$.

Show that $E[(Y - E(Y))^3] = \lambda$ (5mks)

c) (i) Give 3 conditions for a binomial model. (3mks)

Assume that on average one telephone number out of 15 is busy. What is the probability that if 6 randomly selected telephone numbers are called.

(ii) 3 will be busy (2mks)

(iii) Not more than 3 will be busy (2mks)

(iv) At least three of them will be busy (2mks)

d) A random variable X has probability density function $f(x)$ is given by

$f(x) = ce^{-2x}, 0 < x < \infty$

(i) Find moment generating function mgf of X. (2mks)

(ii) Show that the mean is $\frac{1}{2}$ and variance is $\frac{1}{4}$. (3mks)

Question 2(20 Marks)

(a) (i) Suppose that the random variable X has moment generating function $M_X(t)$. For arbitrary constants a and b, show that mgf of $ax + b$ is $e^{bt}\ M_X(at)$ (3mks)

Suppose that the random variable X follows a normal distribution with expected value μ and variance σ^2.

(ii) Show that X has mgf (7mks)

$$M_X(t) = \exp\{\mu t + \sigma^2 t^2\}$$

(iii) Using (i) and (ii), find the mgf of the random variable.

$$Z = \frac{x-\mu}{\sigma}$$ (3mks)

(b) The number of calories in a salad on the lunch menu is normally distributed with mean μ =200 and standard deviation σ =5. Find the probability that the salad you select will contain:

(i) More than 208 calories (2mks)

(ii) Exactly 200 calories (2mks)

(iii) Between 190 and 200 calories (3mks)

Question 3(20 Marks)

a) The continuous random variable X follows the gamma distribution with probability density function

$$f(x) = \frac{\beta^{\alpha}}{\Gamma\alpha} e^{-\beta x} x^{\alpha-1}, x > 0; \alpha, \beta > 0$$

Here α and β are positive constants and $\Gamma\alpha$ denotes the gamma function defined by

$$\Gamma\alpha = \int_0^{\infty} x^{\alpha-1} e^{-x} dx$$

(i) Show that X has the following moment generating function (mgf). (6mks)

$$M_X(t) = \left(\frac{\beta}{\beta - \beta t}\right)^{\alpha}, t < \beta$$

(ii) Hence find the expected value and variance of X . (4mks)

b) The continuous random variable X follows the exponential distribution with probability density function.

$$f(x) = \lambda e^{-\lambda x}, x > 0; \lambda > 0$$

(i) Show that X has moment generating function. (6mks)

$$M_X(t) = \frac{\lambda}{\lambda - t}$$

(ii) Using this result find the expected value and variance of X . (4mks)

Question 4 (20 Marks)

a) (i) State four assumptions leading to a hypergeometric distribution. (4mks)

During a particular period, a university information technology office received 20 service orders for problems with printers, of which 8 were laser printers and 12 were inkjet models, samples of 5 of these service orders is to be selected for inclusion in a customer satisfaction survey. Suppose that the 5 are selected in a completely random manner, so that any particular subset of size 5 has the same chance of being selected as does any other subset. What is the probability that exactly 2 of the selected service orders were from inkjet printers. (5mks)

b) The probability that a pen drawn at random from a box is defective is 0.1. If a sample of 6 pens is taken, find the probability that it will contain:

(i) No defective pen (2mks)

(ii) 5 to 6 defective pens (2mks)

(iii) More than 2 defective pens (2mks)

(iv) Less than 3 defective pens (2mks)

c) A car hire firm has 2 cars which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson variate with mean 1.5. Calculate the proportion of days on which some demands is refused. (3mks)