MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS

2018/2019 ACADEMIC YEAR

***FOURTH* YEAR *FIRST* SEMESTER**

SCHOOL OF SCIENCE
BACHELOR OF SCIENCE IN APPLIED STATISTICS WITH COMPUTIONG

COURSE CODE: STA 419

COURSE TITLE: INTRODUCTION TO MEASURE AND PROBABILITY

DATE: DEC 2018 **TIME:**

INSTRUCTIONS TO CANDIDATES

1. Answer Question **ONE** and any other **TWO** questions
2. Show all your working and be neat
3. Do not write on the question paper

*This paper consists of **FOUR** printed pages. Please turn over.*

QUESTION ONE (30 MARKS)

a) What do you understand by the following terms

i). Field (2marks)

ii). σ – Algebra (2marks)

iii). Borel σ – Algebra (2marks)

iv). Measure (2marks)

v). Probability measure (2marks)

b) Let $\mathcal{F}_1$ $\mathcal{F}_2$………be a sequence of collections of subsets of Ω, such that $\mathcal{F}_n \subseteq \mathcal{F}_{n+1}$ for each n

i). Suppose that each F_1 is an algebra. Prove that $\bigcup_{i=1}^{\infty} \mathcal{F}_i$ is also an algebra (3marks)

ii). Suppose that each $\mathcal{F}_1$ is algebra. Show (by counter example) that $\bigcup_{i=1}^{\infty} \mathcal{F}_i$ might not be σ – Algebra (3marks)

c) Let ($\Omega_1, \mathcal{F}_1, P_1$) be Lebesque measure on [0; 1]. Consider a second probability triple ($\Omega_2, \mathcal{F}_2, P_2$) defined as follows: Ω_2=(1,2), $\mathcal{F}_2$ consists of all subsets Ω_2 and ,P_2 is defined by ,$P_2\{1\}=\frac{1}{3}$,,$P_2\{2\}=\frac{2}{3}$ and additivity. Let ($\Omega, \mathcal{F}, P$) be the product measure of ($\Omega_1, \mathcal{F}_1, P_1$) and ($\Omega_2, \mathcal{F}_2, P_2$) .

i). Express each of Ω, $\mathcal{F}$ and P as explicitly as possible (3marks)

ii). Find a set $A \subseteq \mathcal{F}$ such that $P(A)=\frac{3}{4}$ (3marks)

d) What does the following statements mean

i). Converge almost surely (2 marks)

ii). Converge almost everywhere (2 marks)

iii). Converge in Probability (2 marks)

iv). Converge in r^{th} mean (2 marks)

QUESTION TWO (20 MARKS)

The following theorem describes the relationship among all the convergence modes. Prove each of them

i). If $X_n \xrightarrow{a.s} X$ then $X_n \xrightarrow{p} X$ (2marks)

ii). If $X_n \xrightarrow{p} X$, then $X_{nk} \xrightarrow{a.s} X$ for some subsequence X_{nk} (3marks)

iii). If $X_n \xrightarrow{r} X$, then $X_n \xrightarrow{p} X$ (2marks)

iv). If $X_n \xrightarrow{p} X$ and $|X_n|^r$ is uniformly integrable, then $X_n \xrightarrow{r} X$ (5marks)

v). If $X_n \xrightarrow{p} X$, and lim $\sup_n E|X_n|^r \leq E|X_n|^p$, then $X_n \xrightarrow{r} X$ (4marks)

vi). If $X_n \xrightarrow{p} X$, than $X_n \xrightarrow{d} X$ (4marks)

QUESTION THREE (20 MARKS)

a) Let $(A_n)_n \in N$ be a sequence of events from the probability space $(\Omega, \mathcal{F}, P)$ (Borel-Cantelli Lemma). Prove that

   i). If $\sum_{n=1}^{\infty} P(A_n) < \infty$, then $P(\limsup_{n\to\infty} A_n) = 0$ (4marks)

   ii). If $(A_n)_n \in N$ is independent and $\sum_{n \varepsilon N}^{\infty} P(A_n) = \infty$, then $P(\limsup_{n\to\infty} A_n) = 1$ (6marks)

b) Prove Weak Law of large number, if $X_1, X_2, ...X_n$ are IID with mean $\mu(so, E[|X|] < \infty, and, \mu = E[X])$ then $\bar{X}_n \xrightarrow{p} \mu$ (10 marks)

QUESTION FOUR 20 MARKS

a) Prove Strong Law of large number, if $X_1, X_2, ...X_n$ are IID with mean then μ

$\bar{X}_n \xrightarrow{a.s} \mu$ (10marks)

b) Prove Radon- Nikodym theorem i.e. Let $(\Omega, \mathcal{F}, P)$ be $\sigma-$finite measure space, and let v be a measurable on $(\Omega, \mathcal{F})$ with $v << \mu$. Then there exists a measurable function $X \geq 0$ such that $v(A) = \int_A X d\mu$ for all A$\in \mathcal{F}$. X is unique in the sense that if another measurable function Y also satisfies the equation, then $X = Y$

(10marks)

******* END *******