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MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2017/2018 ACADEMIC YEAR
FOURTH YEAR SECOND SEMESTER

SCHOOL OF SCIENCE
BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION (SCIENCE)

COURSE CODE: PHY 415
COURSE TITLE: STATISTICAL MECHANICS

DATE: 16TH APRIL 2018 **TIME: 0830 – 1030AM**

INSTRUCTIONS TO CANDIDATES

1. Answer Question **ONE** and any other **TWO** questions
2. *Question one carries 30 marks while each of the others carries 20 marks.*
3. *Credit will be awarded for clear explanations and illustrations.*

*This paper consists of **4** printed pages. Please turn over.*

The following are useful constants and formulae

(i) $\ln N! = N \ln W - N$ *and* $N! = \left(\frac{N}{e}\right)^N$

(ii) $\int e^{-ax^2} = \sqrt{\frac{\pi}{a}}$

(iii) $\ln(1 \pm x) = 1 \pm x \pm \frac{1}{2}x^2 + ...$

(iv) $\int \frac{dx}{e^x + 1} = -\ln(1 + e^x)$

(v) volume of hypersphere, $V_a = \frac{\pi f}{\left(\frac{1}{2}f\right)!}$ where f is degrees of freedom.

(v) Planck's constant, $h = 6.63 \times 10^{-34} J.s$

(vii) massofanelectron, $M_e = 9.11 \times 10^{-31} kg$

$(viii)$ Boltzmann's constant, $K_B = 1.38 \times 10^{-23} JK^{-1}$

Question One: [30marks]

a) Briefly explain the following in statistical mechanics.

i. Macro state (2marks)

ii. Microstate (2marks)

iii. Phase Space. (2marks)

iv. An ensemble (2marks)

b) Briefly explain the term partition function Z for a statistical system. [4 Marks]

c) Distinguish between canonical and grand canonical ensembles. [4 Marks]

d) Give Boltzmann's statistical definition of entropy and present its meaning [3 Marks]

e) State the basic differences in the fundamental assumptions underlying Maxwell-Boltzmann (MB) and Fermi-Dirac(FD) statistics. [3Marks]

f) The entropy of an ideal gas of a micro canonical system is defined as $S(E, V) = NK \ln\left[V(\frac{4\pi ME}{3h^2N})^{\frac{3}{2}}\right] + \frac{3NK}{2}$. Show that the absolute temperature is given by $T = \frac{2U}{3NK}$ [5 Marks]

g) State the second law of thermodynamics (3marks)

h) Two particles are to be distributed in an energy level, which is 3 fold-degenerate. Find the possible microstates if the particles are indistinguishable bosons (2marks)

Question Two: [20]

a) State two differences between Classical and Quantum Statistics [4marks]

b) Show that the average number of particles, $\overline{n_l}$ occupying a state with degeracy g_i for the indistinguishable bosons particles is given by: $\overline{n_l} = \frac{g_i}{e^{\alpha+\beta\epsilon i}-1}$, where α, ϵ and β are constants. [7 Marks]

c) State the Postulate of equal a priori probabilities in statistical mechanics (3marks)

d) The grand partition function $Z = \sum_{N=0}^{\infty} Z^N Q(V,T)$ where $Q_N(V,T) = \frac{1}{N!}\left[\frac{V}{h^3}(2\pi mKT)^{\frac{3}{2}}\right]^N$. Show that the average number, $\overline{N}$, of particles in the ensemble is given by $\overline{N} = Z\frac{\partial}{\partial Z}\ln Z(Z,V,T)$. (6marks)

Question Three: [20]

a) When do we apply Bose-Einstein statics? [2 Marks]

b) Briefly explain the three distribution laws used to describe the statistical properties of a system namely: Boltzmann statistics, Fermi statistics and Bose statistics. [9marks]

c) A system consisting of 6 fermions has total energy 6 units. These particles are to be distributed in 5 energy levels $\varepsilon_0 = 0$, $\varepsilon_1 = 1$, $\varepsilon_2 = 2$, $\varepsilon_3 = 3$, $\varepsilon_4 = 4$ units. Each energy level is triply degenerate. Find the possible microstates. [6marks]

d) Explain briefly ergodic hypothesis (3marks)

Question Four: [20]

a) State Clausius statement on thermodynamics. (2marks)

b) Briefly explain the three principal ensembles of statistical thermodynamics [6marks]

c) Use Helrnholtz free energy equation $A(V,T) = -KT\ln Q_N(V,T)$ where

$Q_N(V,T) = \frac{1}{N!}\left[\frac{V}{h^3}(2\pi mKT)^{\frac{3}{2}}\right]^N$ to show that Pressure $P = \frac{NKT}{V}$ and total internal energy is given as $U = \frac{3NKT}{2}$ (12marks]

End//

MARKING SCHEME PHY 450

QN 0NE

a) (i) A Microstate is defined as a state of the system where all the parameters of the constituents (particles) are specified. Many microstates exist for each state of the system specified in macroscopic variables(E, V, N, ...) and there are many parameters for each state.

(ii) A macrostate is defined as a state of the system where the distribution of particles over the energy levels is specified. The macrostate includes what are the different energy levels and the number of particles having particular energies. It contains many microstates.

(iii) A phase space is a space in which all possible states of a system are represented, with each possible state of the system corresponding to one unique point in the phase space. For mechanical systems, the phase space usually consists of all possible values of position and momentum variables

(iv) An ensemble (also statistical ensemble or thermodynamic ensemble) is an idealization consisting of a large number of mental copies (sometimes infinitely many) of a system, considered all at once, each of which represents a possible state that the real system might be in.

b) Partition function Z describes the statistical properties of a system in thermodynamic equilibrium. It is a function of temperature and other parameters, such as the volume enclosing a gas. Most of the aggregate thermodynamic variables of the system, such as the total energy, free energy, entropy, and pressure, can be expressed in terms of the partition function or its derivatives.

c) Canonical ensemble or NVT ensemble -- an ensemble of systems, each of which can share its energy with a large heat reservoir or heat bath. The system is allowed to exchange energy with the reservoir, and the heat capacity of the reservoir is assumed to be so large as to maintain a fixed temperature for the coupled system.

Grand canonical ensemble -- an ensemble of systems, each of which is again in thermal contact with a reservoir. But now in to energy, there is also exchange of particles. The temperature is still assumed to be fixed.

d) $S = K\ln\Omega$, where Ω is the number of microscopic states of the system. Physically entropy is measurement of the disorder of a system.

e) Maxwell-Boltzmann (MB) and Fermi-Dirac (FD) statistics. : -FD , as compared to MB , statistics has two additional assumptions

-the principle of in distinguishability; Identical particles cannot be distinguished from one another

-Pauli's exclusion principle; Not more than one particle can occupy a quantum state. In the limit of non-degeneracy, FD statistics gradually becomes MB statistics

f) The entropy of an ideal gas of a micro canonical system is defined as $S(E,V) = NK\ln\left[V\left(\frac{4\pi ME}{3h^2N}\right)^{\frac{3}{2}}\right] + \frac{3NK}{2}$. Show that the absolute temperature is given by $T = \frac{2U}{3NK}$

- Internal energy $U(S,V) = \left[\left(\frac{3h^2}{4\pi m}\right)\right]\frac{N}{V^{2/3}}e^{\left(\frac{2S}{3NK}-1\right)}$
- From the relation of energy , the temperature is given as: $T = \left(\frac{\partial U}{\partial S}\right)_V$

$$T = \left(\frac{2U}{3NK}\right).$$

g) The second law of thermodynamics states that the entropy of an isolated system never decreases, because isolated systems spontaneously evolve towards thermodynamic equilibrium—the state of maximum entropy

h) If the particles are indistinguishable bosons, the number of possible microstates is

$$\Omega = \frac{(ni+gi-1)!}{ni!(gi-1)!} \qquad = \frac{4!}{2!2!} = 6$$

QN TWO

a) State two differences between Classical and Quantum Statistics [4marks]:

- If molecules, atoms, or subatomic particles are in the liquid or solid state, the Pauli Exclusion Principle prevents two particles with identical wave functions from sharing the same space.
- There is no restriction on particle energies in classical physics.

-There are only certain energy values allowed in quantum systems.

b)

c) Postulate of equal a priori probabilities. This postulate applies to the micro canonical (EVN) ensemble. Simply put, it asserts that the weighting function π is a constant in the micro canonical ensemble. All microstates of equal energy are accorded the same weight.

d)

QN THREE

a) - B-E statistics is applicable to the system of identical, indistinguishable particles, which have integral spin (0, 1, 2….). Particles with integral spin are called bosons

b) Briefly explain the three distribution laws used to describe the statistical properties of a system. [9]

(i) MAXWELL-BOLTZMANN (M-B) STATISTICS M-B statistics is applicable to the system of identical, distinguishable particles. The particles are so far apart that they are distinguishable by their position. In the language of quantum mechanics, the application of classical statistics is valid if the average separation between particles is much greater than the average de Broglie wavelength of the particle. In this situation the wave functions of the particles don't overlap. The particle may have any spin. The classical statistics put no restriction on the number of particles that occupy a state of the system. M-B statistics can be safely applied to dilute gases at room and higher temperature.

(ii) BOSE-EINSTEIN (B-E) STATISTICS B-E statistics is applicable to the system of identical, indistinguishable particles, which have integral spin (0, 1, 2….). Particles with integral spin are called bosons. Bosons don't obey Pauli's exclusion principle. So any number of bosons can occupy a single quantum state. The particles are close enough so that their wave functions overlap. Examples of bosons are photons (spin 1), phonons (quantum of acoustical vibration), pions, alpha particles, helium atoms etc.

(iii) FERMI-DIRAC (F-D) STATISTICS F-D statistics is applicable to the system of identical, indistinguishable particles, which have odd-half-integral spin (1/2, 3/2,5/2…). Particles with odd-half-integral spin are called fermions and they obey Pauli's exclusion principle. Hence, not more than one fermion can occupy a quantum state. The F-D statistics is valid if the average separation between fermions is comparable to the average de Broglie wavelength of fermions so that their wave functions overlap. Examples of fermions are electrons, positrons, µmesons, protons, neutrons etc. In the limit of high temperature and low particle density the two quantum statistics (B-E and F-D) yield results identical to those obtained using the classical statistics c)

Sol. The possible macrostates are 5. They are:{3,2,0,0,1}, {3,1,1,1,0}, {2,3,0,1,0}, {3,0,3,0,0}, {2,2,2,0,0}. The thermodynamic probability (the number of microstates) of a macrostate is given by

$$\Omega = \prod_i \frac{g_i!}{n_i!(g_i - n_i)!}$$

- ✓ The number of microstates associated with first macrostate is

$$\Omega_i = \frac{3_i!}{3!(3-3)!} \cdot \frac{3_i!}{2!(3-2)!} \cdot \frac{3_i!}{0!(3-0)!} \cdot \frac{3_i!}{0!(3-0)!} \cdot \frac{3_i!}{3!(3-1)!} = 9$$

- ✓ Similarly, the number of microstates associated with other macrostates can be calculated. Theycome out to be Ω_{II} = 27, Ω_{III} = 9, Ω_{IV} = 1, Ω_V = 27.
- ✓ The thermodynamic probability of the system is $\Omega = \Sigma\, \Omega_i = 9 + 27 + 9 + 1 + 27 = 73$

d) Ergodic hypothesis says that, over long periods of time, the time spent by a particle in some region of the phase space of microstates with the same energy is proportional to the volume of this region, i.e., that all accessible microstates are equiprobable over a long period of time.

QN FOUR

a) Clausius statement - "Heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time." This may be restated as " No process is possible whose sole result is the transfer of heat from a colder to a warmer body without some other change

b) Briefly explain the principal ensembles of statistical thermodynamics. [6]
- Different macroscopic environmental constraints lead to different types of ensembles, with particular statistical characteristics. The following are the most important:

-Microcanonical ensemble or NVE ensemble -- an ensemble of systems, each of which is required to have the same total energy (i.e. thermally isolated).
-Canonical ensemble or NVT ensemble -- an ensemble of systems, each of which can share its energy with a large heat reservoir or heat bath. The system is allowed to exchange energy with the reservoir, and the heat capacity of the reservoir is assumed to be so large as to maintain a fixed temperature for the coupled system.
-Grand canonical ensemble -- an ensemble of systems, each of which is again in thermal contact with a reservoir. But now in addition to energy, there is also exchange of particles. The temperature is still assumed to be fixed.

c)

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