MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2017/2018 ACADEMIC YEAR
THIRD YEAR FIRST SEMESTER

SCHOOL OF SCIENCE
BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION (SCIENCE)

COURSE CODE: PHY 310
COURSE TITLE: MATHEMATICAL PHYSICS

DATE: 26TH APRIL, 2018 **TIME: 0830 - 1030HRS**

INSTRUCTIONS TO CANDIDATES

1. Answer Question **ONE** and any other **TWO** questions
2. *Question one carries 30 marks while each of the others carries 20 marks.*
3. *Credit will be awarded for clear explanations and illustrations.*

This paper consists of 3 printed pages. Please turn over.

Standard Laplace transforms

f(t)	$L\{f(t)\}$
1	$1/_s$
e^{at}	$1/_{s-a}$
t^n	$n!/_{s^{n+1}}$

QUESTION ONE

a) Distinguish between a scalar field and a vector field (2mks)

b) The vector field F is defined by $\boldsymbol{F} = 2xz\boldsymbol{i} + 2yz\boldsymbol{j} + (x^2 + 2y^2 z)\boldsymbol{k}$. Calculate $\nabla \times \mathbf{F}$ and deduce that F can be written $\mathbf{F} = \nabla\varphi$. Determine the form of φ (5mks)

c) Verify by direct calculation that $\nabla.(\mathbf{a} \times \mathbf{b}) = \mathbf{b}.(\nabla \times \mathbf{a}) - \mathbf{a}.(\nabla \times \mathbf{b})$ (4mks)

d) Determine the Laplace transform of $t^2 e^t$ (3mks)

e) A radioactive isotope decays in such a way that the number of atoms present at a given time, $N(t)$, obeys the equation: $\frac{dN}{dt} = -\lambda N$. If there are initially N_o atoms present, find $N(t)$ at later times. (5mks)

f) The acceleration of a particle at any time $t \geq 0$ is given $\boldsymbol{a} = \frac{dv}{dt} = 12\cos 2t\boldsymbol{i} - 8\sin 2t\,\boldsymbol{j} + 16t\boldsymbol{k}$. If the velocity **v** and displacement **r** are zero at $t = 0$, find v and r at any time (6mks)

g) The voltage from a square wave generator is of the form $v(t) = \begin{cases} 0, -4 < t < 0 \\ 10, 0 < t < 4 \end{cases}$ and has a period of 8ms. Find the Fourier series for this periodic function (5mks)

QUESTION TWO

a) Verify Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2 dy$ where C is a closed curve of the region bounded by $y = x$ and $y = x^2$ (5marks)

b) The current flowing in an electrical circuit is given by the differential equation $Ri + L\frac{di}{dt} = E$ where E, L and R are constants. Use Laplace transforms to solve the equation for current *i* given that when t=0, *i*=0 (8mks)

c) Find the total work done in moving a particle in a force field given by $\boldsymbol{F} = 3xy\boldsymbol{i} - 5z\boldsymbol{j} + 10x\boldsymbol{k}$ along the curve $x = t^2 + 1, y = 2t^3, z = t^3$ from t=1 to t=2 (4mks)

d) Determine the constant a so that the vector $\mathbf{v} = (x + 3y)\mathbf{i} + (y - 2z)\mathbf{j} + (x + az)\mathrm{k}$ is solenoidal (3marks)

QUESTION THREE

a) If E and $\varphi$are the electric field strength and the electric potential respectively then $E = -\mathbf{grad}\varphi$ and $\mathbf{div}\, E = \frac{\rho}{\varepsilon}$. Find the Poisson's equation (4marks)

b) Determine the solution of the Laplace's equation in Cartesian coordinate (6mks)

c) A metal bar, insulated a long its sides is 1 m long . it is initially room temperature of $15°\mathrm{C}$ and at time t=0, the ends are placed into ice at$0°\mathrm{C}$.Find an expression for the temperature at a point P at distance x m from one end at any time t seconds after t=0 (10mks)

QUESTION FOUR

a) Solve the differential equation $2\frac{d^2y}{dx^2} - 11\frac{dy}{dx} + 12y = 3x - 2$ (6marks)

b) Show that the force $\mathrm{F} = (2xy + z^3)\mathbf{i} + x^2\mathbf{j} + 3xz^2\,\mathbf{k}$ is a conservative force field (5marks)

c) Find the corresponding scalar potential function force in this field (5mks)

d) Find the work done in moving an object in this field from $p_1(1,-2,1)$ to $p_2(3,1,4)$ (4mks)

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