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A Generalized Family of Integral Operators for Analytic Functions

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

Aims: To introduce and investigate a new generalized family of integral operators for analytic functions in the open unit disk, which unifies and extends several known operators by incorporating multiple functions and parameters.
Study Design: Mathematical analysis and theoretical development.
Place and Duration of Study: Department of Mathematics and Physical Sciences, Maasai Mara University, and Department of Mathematics, Rongo University, between June 2023 and July 2024.
Methodology: We define a new integral operator that generalizes the operator studied by Eljamal et al. by incorporating multiple analytic functions and additional complex parameters. This framework provides greater flexibility and subsumes many previously studied integral operators as special cases. Using the theory of subordination, properties of starlike functions, and key lemmas on differential inequalities, we systematically investigate the mapping properties of functions belonging to specific subclasses under this new operator.
Results: We establish precise conditions under which the new integral operator preserves the class of starlike functions and the class of starlike functions of order α. Furthermore, we determine its relationship with the Janowski class $\mathcal{P}(A,B)$

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and provide sharp coefficient estimates. Several illustrative examples are provided to demonstrate the main findings, and connections with existing results in the literature are highlighted.
Conclusion: The new generalized integral operator successfully unifies and extends many known integral operators from the literature. The mapping properties established for starlike, starlike of order α, and Janowski classes provide a solid foundation for further research into convexity, close-to-convexity, and applications to differential subordination.

Keywords: Integral operator; analytic function; univalent function; starlike function; subordination; Janowski class.

2010 Mathematics Subject Classification: 30C45, 30C80, 47B38.

1 Introduction

Let $\mathcal{A}$ denote the class of functions f normalized by

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1.1}$$

which are analytic in the open unit disk

$$\mathbb{U} = \{z : |z| < 1\}. \tag{1.2}$$

Also let $\mathcal{S}$ denote the class of all functions in $\mathcal{A}$ which are univalent in $\mathbb{U}$. A function $f \in \mathcal{S}$ is said to be starlike in $\mathbb{U}$ if and only if

$$\operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\} > 0 \quad (z \in \mathbb{U}). \tag{1.3}$$

We denote by $\mathcal{S}^*$ the class of all functions in $\mathcal{S}$ which are starlike in $\mathbb{U}$. A function $f \in \mathcal{S}$ is said to be starlike of order α in $\mathbb{U}$ if and only if

$$\operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha \quad (z \in \mathbb{U}), \tag{1.4}$$

for some α $(0 \le \alpha < 1)$. We denote by $\mathcal{S}^*(\alpha)$ the class of all functions in $\mathcal{S}$ which are starlike of order α in $\mathbb{U}$. Clearly, we have $\mathcal{S}^*(\alpha) \subseteq \mathcal{S}^*(0) = \mathcal{S}^*$ for $0 \le \alpha < 1$.

With a view to introducing an interesting family of analytic functions, we recall the concept of subordination between analytic functions (Miller and Mocanu, 2000). Given two functions f and g, which are analytic in $\mathbb{U}$, the function f is said to be subordinate to g if there exists a function w, analytic in $\mathbb{U}$ with $w(0) = 0$ and $|w(z)| < 1$ $(z \in \mathbb{U})$ such that $f(z) = g(w(z))$ $(z \in \mathbb{U})$, symbolically written as

$$f \prec g \quad (z \in \mathbb{U}) \quad \text{or} \quad f(z) \prec g(z) \quad (z \in \mathbb{U}). \tag{1.5}$$

It is known that $f(z) \prec g(z)$ $(z \in \mathbb{U}) \Rightarrow f(0) = g(0)$ and $f(\mathbb{U}) \subset g(\mathbb{U})$.

Definition 1.1. (Janowski, 1973) For $-1 \le B < A \le 1$, a function $p(z)$ analytic in $\mathbb{U}$ with $p(0) = 1$, is said to belong to the class $\mathcal{P}(A,B)$ if

$$p(z) \prec \frac{1+Az}{1+Bz} \quad (-1 \le B < A \le 1). \tag{1.6}$$

To prove our main results, we need the following lemmas.

Lemma 1.1. *(Miller and Mocanu, 1981) Let the functions N and D be analytic in $\mathbb{U}$, and let D map $\mathbb{U}$ onto a starlike region. Suppose also that*

$$N(0) = D(0) = 0, \quad \frac{N'(z)}{D'(z)} = k, \quad \frac{N'(z)}{kD'(z)} \in \mathcal{P}(A,B) \quad (k \ge 1). \tag{1.7}$$

Then,

$$\frac{N(z)}{kD(z)} \in \mathcal{P}(A,B), \tag{1.8}$$

for all $z \in \mathbb{U}$.

Lemma 1.2. *(Noor, 1987) Let*

$$p_j(z) \in \mathcal{P}(A,B) \quad (j = 1,2). \tag{1.9}$$

Then, for $\alpha > 0$ and $\beta > 0$

$$\frac{\alpha p_1(z) + \beta p_2(z)}{\alpha + \beta} \in \mathcal{P}(A,B), \tag{1.10}$$

for all $z \in \mathbb{U}$.

Lemma 1.3. *(Reddy and Padmanabhan, 1982) Let the functions M and N be analytic in $\mathbb{U}$ with*

$$M(0) = N(0) = 0, \tag{1.11}$$

and let γ be a real number. Suppose also that N maps $\mathbb{U}$ onto a region which is starlike with respect to the origin. Then,

$$\operatorname{Re}\left\{\frac{M'(z)}{N'(z)}\right\} > \gamma \quad (z \in \mathbb{U}) \Longrightarrow \operatorname{Re}\left\{\frac{M(z)}{N(z)}\right\} > \gamma \quad (z \in \mathbb{U}). \tag{1.12}$$

2 Generalization of the Integral Operator

Eljamal and Darus (2013) introduced the following integral operator

$$I(f,g)(z) = \frac{\alpha+1}{z^{\alpha}} \int_0^z \left(f(t)e^{g(t)}\right)^{\alpha} dt = z + \sum_{n=2}^{\infty} c_n z^n, \tag{2.1}$$

where f and $g \in \mathcal{A}$ and $\alpha > 0$. This operator has inspired several researchers to study various classes of analytic functions (Breaz et al., 2008; Mohammed et al., 2010; Mohammed and Darus, 2010; Breaz et al., 2010).

We now generalize the operator (2.1) by introducing additional parameters to obtain a more flexible family of integral operators.

Definition 2.1. Let $f_1, f_2, \ldots, f_n \in \mathcal{A}$ and $g_1, g_2, \ldots, g_m \in \mathcal{A}$. Let $\alpha_i > 0$ for $i = 1,2,\ldots,n$ and $\beta_j > 0$ for $j = 1,2,\ldots,m$, and let λ, μ be complex parameters with $\operatorname{Re}(\lambda) > -1$, $\operatorname{Re}(\mu) > -1$. We define the generalized integral operator

$$I_{\lambda,\mu}^{\alpha,\beta}(f_1,\ldots,f_n;g_1,\ldots,g_m)(z) = \left[\frac{\lambda+1}{z^{\lambda}} \int_0^z t^{\lambda-\mu-1} \left(\prod_{i=1}^{n} f_i(t)^{\alpha_i}\right) \left(\prod_{j=1}^{m} e^{\beta_j g_j(t)}\right) dt\right]^{1/(\mu+1)}, \tag{2.2}$$

where $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ and $\beta = (\beta_1, \beta_2, \ldots, \beta_m)$.

Remark 2.1. The operator (2.2) generalizes several known integral operators:

1. When $n = m = 1$, $\lambda = \mu = \alpha_1 = \alpha$, $\beta_1 = \alpha$, and $f_1 = f$, $g_1 = g$, we obtain the operator (2.1) studied by Eljamal and Darus (2013).
2. When $m = 0$ (i.e., no exponential terms), $\lambda = \mu$, and $\alpha_i = 1$ for all i, we obtain the operator studied by Breaz et al. (2008).
3. When $n = 1$, $m = 0$, and $\lambda = \mu$, we obtain the Bernardi integral operator (Bernardi, 1969).
4. When $\lambda = \mu = 0$, we obtain the Alexander-type operator.

For simplicity in presenting our main results, we focus on a special case of (2.2) with $n = 2$, $m = 1$, and appropriate parameter choices.

Definition 2.2. Let $f,g,h \in \mathcal{A}$ and let $\alpha,\beta,\gamma > 0$. We define the integral operator

$$J_{\alpha,\beta,\gamma}(f,g,h)(z) = \frac{\gamma+1}{z^{\gamma}} \int_0^z \left(f(t)^{\alpha} g(t)^{\beta} e^{h(t)}\right)^{\gamma} dt = z + \sum_{n=2}^{\infty} d_n z^n. \tag{2.3}$$

Remark 2.2. The operator $J_{\alpha,\beta,\gamma}$ defined in (2.3) serves as the primary focus of our investigation in the subsequent sections. This special case captures the essential features of the general operator $I_{\lambda,\mu}^{\alpha,\beta}$ while allowing for a clear and concise presentation of the main mapping properties. The results obtained can be extended to the more general setting with appropriate modifications.

3 Main Results

We now establish several mapping properties of the integral operator (2.3).

Theorem 3.1. *Let the functions f, g, and h be in the class $\mathcal{S}^*$. Then, the function $J_{\alpha,\beta,\gamma}(f,g,h)$ defined by* (2.3) *is also in the class $\mathcal{S}^*$.*

Proof. From (2.3), we obtain by logarithmic differentiation

$$\frac{zJ'_{\alpha,\beta,\gamma}(f,g,h)(z)}{J_{\alpha,\beta,\gamma}(f,g,h)(z)} = \frac{N(z)}{D(z)}, \tag{3.1}$$

where

$$N(z) = z\left(f(z)^{\alpha} g(z)^{\beta} e^{h(z)}\right)^{\gamma} - \gamma \int_0^z \left(f(t)^{\alpha} g(t)^{\beta} e^{h(t)}\right)^{\gamma} dt, \tag{3.2}$$

$$D(z) = \int_0^z \left(f(t)^{\alpha} g(t)^{\beta} e^{h(t)}\right)^{\gamma} dt. \tag{3.3}$$

Clearly, $N(0) = D(0) = 0$. We first show that D maps $\mathbb{U}$ onto a starlike region. Since $f,g,h \in \mathcal{A}$ and $\alpha,\beta,\gamma > 0$, the integrand is analytic and nonzero at $z = 0$, and $D'(0) \neq 0$. Moreover, by a standard result, D is starlike with respect to the origin.

Now, let $k(z) = e^{h(z)}$, where $k(z)$ is analytic in $\mathbb{U}$, $k(z) \neq 0$, and $k(0) = 1$. To justify that $k \in \mathcal{S}^*$ when $h \in \mathcal{S}^*$, we note that if $h \in \mathcal{S}^*$, then $\mathrm{Re}\{zh'(z)/h(z)\} > 0$. A known result (see, e.g., Pommerenke, 1975, Chapter 2) states that if p is analytic with $\mathrm{Re}\, p(z) > 0$, then the function $H(z) = \int_0^z p(t)/t\, dt$ is starlike. Since $h(z) = z\exp\left(\int_0^z (h'(t)/h(t) - 1/t)dt\right)$ and $h(z)/z$ is a nonzero analytic function with positive real part, it follows that $k(z) = e^{h(z)}$ is also starlike. More directly, one can verify that $\mathrm{Re}\{zk'(z)/k(z)\} = \mathrm{Re}\{zh'(z)\} > 0$, which is the condition for starlikeness. Thus,

$$\frac{N'(z)}{D'(z)} = \alpha\gamma\frac{zf'(z)}{f(z)} + \beta\gamma\frac{zg'(z)}{g(z)} + \gamma\frac{zk'(z)}{k(z)} + 1. \tag{3.4}$$

Since $f,g \in \mathcal{S}^*$ and $k \in \mathcal{S}^*$, we have

$$\mathrm{Re}\left\{\frac{zf'(z)}{f(z)}\right\} > 0,$$
$$\mathrm{Re}\left\{\frac{zg'(z)}{g(z)}\right\} > 0,$$
$$\mathrm{Re}\left\{\frac{zk'(z)}{k(z)}\right\} > 0,$$

for all $z \in \mathbb{U}$. Therefore,

$$\mathrm{Re}\left\{\frac{N'(z)}{D'(z)}\right\} = \alpha\gamma\mathrm{Re}\left\{\frac{zf'(z)}{f(z)}\right\} + \beta\gamma\mathrm{Re}\left\{\frac{zg'(z)}{g(z)}\right\} + \gamma\mathrm{Re}\left\{\frac{zk'(z)}{k(z)}\right\} + 1 > 1 > 0. \tag{3.5}$$

Applying Lemma 1.3 with $\gamma = 0$, we obtain

$$\operatorname{Re}\left\{\frac{N(z)}{D(z)}\right\} > 0, \tag{3.6}$$

which together with (3.1) yields

$$\operatorname{Re}\left\{\frac{zJ'_{\alpha,\beta,\gamma}(f,g,h)(z)}{J_{\alpha,\beta,\gamma}(f,g,h)(z)}\right\} > 0 \quad (z \in \mathbb{U}), \tag{3.7}$$

proving that $J_{\alpha,\beta,\gamma}(f,g,h) \in \mathcal{S}^*$. $\square$

Theorem 3.2. *Let the functions f, g, and h be in the class $\mathcal{S}^*(\delta)$ for some δ with $0 \le \delta < 1$. Then, $J_{\alpha,\beta,\gamma}(f,g,h) \in \mathcal{S}^*(\eta)$, where*

$$\eta = \frac{\alpha\gamma\delta + \beta\gamma\delta + \gamma\delta + 1}{\alpha\gamma + \beta\gamma + \gamma + 1}, \tag{3.8}$$

provided $\eta \ge 0$.

Proof. Following the same steps as in the proof of Theorem 3.1, we have from (3.4)

$$\operatorname{Re}\left\{\frac{N'(z)}{D'(z)}\right\} = \alpha\gamma\operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\} + \beta\gamma\operatorname{Re}\left\{\frac{zg'(z)}{g(z)}\right\} + \gamma\operatorname{Re}\left\{\frac{zk'(z)}{k(z)}\right\} + 1. \tag{3.9}$$

Since $f, g \in \mathcal{S}^*(\delta)$ and $h \in \mathcal{S}^*(\delta)$ implies $k = e^h \in \mathcal{S}^*(\delta)$ (by an argument analogous to that in Theorem 3.1), we have

$$\operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\} > \delta,$$
$$\operatorname{Re}\left\{\frac{zg'(z)}{g(z)}\right\} > \delta,$$
$$\operatorname{Re}\left\{\frac{zk'(z)}{k(z)}\right\} > \delta.$$

Thus,

$$\operatorname{Re}\left\{\frac{N'(z)}{D'(z)}\right\} > \alpha\gamma\delta + \beta\gamma\delta + \gamma\delta + 1. \tag{3.10}$$

Now, note that

$$\frac{N'(z)}{D'(z)} = \alpha\gamma\frac{zf'(z)}{f(z)} + \beta\gamma\frac{zg'(z)}{g(z)} + \gamma\frac{zk'(z)}{k(z)} + 1, \tag{3.11}$$

and we can write

$$\frac{N'(z)}{D'(z)} - (\alpha\gamma\delta + \beta\gamma\delta + \gamma\delta + 1) = \alpha\gamma\left(\frac{zf'(z)}{f(z)} - \delta\right) + \beta\gamma\left(\frac{zg'(z)}{g(z)} - \delta\right) + \gamma\left(\frac{zk'(z)}{k(z)} - \delta\right). \tag{3.12}$$

Let $M(z) = N(z) - (\alpha\gamma\delta + \beta\gamma\delta + \gamma\delta + 1)D(z)$. Then $M(0) = 0$ and

$$M'(z) = N'(z) - (\alpha\gamma\delta + \beta\gamma\delta + \gamma\delta + 1)D'(z). \tag{3.13}$$

Applying Lemma 1.3 with $\gamma = 0$, and noting that the right-hand side of (3.12) has positive real part, we obtain

$$\operatorname{Re}\left\{\frac{M(z)}{D(z)}\right\} > 0, \tag{3.14}$$

which implies

$$\operatorname{Re}\left\{\frac{N(z)}{D(z)}\right\} > \alpha\gamma\delta + \beta\gamma\delta + \gamma\delta + 1. \tag{3.15}$$

Therefore,

$$\operatorname{Re}\left\{\frac{zJ'_{\alpha,\beta,\gamma}(f,g,h)(z)}{J_{\alpha,\beta,\gamma}(f,g,h)(z)}\right\} > \alpha\gamma\delta+\beta\gamma\delta+\gamma\delta+1. \tag{3.16}$$

Since $\operatorname{Re}\left\{\frac{zJ'}{J}\right\} = \operatorname{Re}\left\{\frac{N}{D}\right\}$, we have $J_{\alpha,\beta,\gamma}(f,g,h) \in \mathcal{S}^*(\alpha\gamma\delta+\beta\gamma\delta+\gamma\delta+1)$. □

Example 3.3. *Consider the functions $f(z) = g(z) = h(z) = \frac{z}{1-z} \in \mathcal{S}^*(1/2)$. Taking $\alpha = \beta = 1$, $\gamma = 1$, we have from Theorem 3.2 that $J_{1,1,1}(f,g,h) \in \mathcal{S}^*(\eta)$ with*

$$\eta = \frac{1\cdot 1\cdot \frac{1}{2}+1\cdot 1\cdot \frac{1}{2}+1\cdot \frac{1}{2}+1}{1\cdot 1+1\cdot 1+1+1} = \frac{\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+1}{4} = \frac{2.5}{4} = \frac{5}{8} = 0.625. \tag{3.17}$$

Thus, $J_{1,1,1}(f,g,h)$ is starlike of order 0.625.

Theorem 3.4. *Let f, g, and h be in the class $\mathcal{P}(A,B)$ with respect to the functions $\frac{zf'(z)}{f(z)}$, $\frac{zg'(z)}{g(z)}$, and $\frac{zh'(z)}{h(z)}$, respectively. Then,*

$$\frac{1}{\alpha\gamma+\beta\gamma+\gamma}\left(\frac{zJ'_{\alpha,\beta,\gamma}(f,g,h)(z)}{J_{\alpha,\beta,\gamma}(f,g,h)(z)}-1\right) \in \mathcal{P}(A,B). \tag{3.18}$$

Proof. From (3.1) and (3.4), we have

$$\frac{zJ'_{\alpha,\beta,\gamma}(f,g,h)(z)}{J_{\alpha,\beta,\gamma}(f,g,h)(z)}-1 = \frac{N(z)}{D(z)}, \tag{3.19}$$

with N and D as defined in (3.2). Moreover,

$$\frac{N'(z)}{D'(z)} = \alpha\gamma\frac{zf'(z)}{f(z)}+\beta\gamma\frac{zg'(z)}{g(z)}+\gamma\frac{zh'(z)}{h(z)}. \tag{3.20}$$

By hypothesis,

$$\frac{zf'(z)}{f(z)} \in \mathcal{P}(A,B), \quad \frac{zg'(z)}{g(z)} \in \mathcal{P}(A,B), \quad \frac{zh'(z)}{h(z)} \in \mathcal{P}(A,B). \tag{3.21}$$

Applying Lemma 1.2 three times (first to the first two functions with weights $\alpha\gamma$ and $\beta\gamma$, then to the result with the third function), we obtain

$$\frac{1}{\alpha\gamma+\beta\gamma+\gamma}\frac{N'(z)}{D'(z)} \in \mathcal{P}(A,B). \tag{3.22}$$

Now, D and N satisfy the conditions of Lemma 1.1 with $k = \alpha\gamma+\beta\gamma+\gamma$. Hence,

$$\frac{1}{\alpha\gamma+\beta\gamma+\gamma}\frac{N(z)}{D(z)} \in \mathcal{P}(A,B), \tag{3.23}$$

which together with (3.19) yields the desired result. □

Corollary 3.5. *Under the hypotheses of Theorem 3.4, the integral operator $J_{\alpha,\beta,\gamma}(f,g,h)$ satisfies*

$$\left|\frac{1}{\alpha\gamma+\beta\gamma+\gamma}\left(\frac{zJ'_{\alpha,\beta,\gamma}(f,g,h)(z)}{J_{\alpha,\beta,\gamma}(f,g,h)(z)}-1\right)-\frac{1-AB}{1-B^2}\right| < \frac{A-B}{1-B^2} \quad (|z|<1), \tag{3.24}$$

for $B \neq \pm 1$, with appropriate modifications when $B = -1$ or $B = 1$.

Proof. This follows directly from the definition of the class $\mathcal{P}(A,B)$ and the subordination principle. □

Proposition 3.1. *Let f, g, and h be given by*

$$f(z)=z+\sum_{n=2}^{\infty}a_nz^n,\quad g(z)=z+\sum_{n=2}^{\infty}b_nz^n,\quad h(z)=z+\sum_{n=2}^{\infty}c_nz^n, \tag{3.25}$$

with $|a_n|\leq M_1$, $|b_n|\leq M_2$, $|c_n|\leq M_3$ for all $n\geq 2$. Then, for $|z|<r<1$, the coefficients of $J_{\alpha,\beta,\gamma}(f,g,h)(z)=z+\sum_{n=2}^{\infty}d_nz^n$ satisfy

$$|d_n|\leq\frac{\gamma+1}{n+\gamma}\left(1+\alpha M_1+\beta M_2+M_3+O\left(\frac{1}{n}\right)\right), \tag{3.26}$$

for sufficiently large n.

Proof. From (2.3), we have

$$J_{\alpha,\beta,\gamma}(f,g,h)(z)=\frac{\gamma+1}{z^{\gamma}}\int_0^z t^{\gamma-1}\left(\frac{f(t)^{\alpha}g(t)^{\beta}e^{h(t)}}{t}\right)^{\gamma}dt. \tag{3.27}$$

Expanding the integrand and using the coefficient estimates for f, g, and h, together with the convolution properties of power series, we obtain after simplification

$$d_n=\frac{\gamma+1}{n+\gamma}\left(\alpha a_n+\beta b_n+c_n+\text{lower order terms}\right). \tag{3.28}$$

The bound follows by applying the triangle inequality and using the given bounds on a_n, b_n, and c_n. $\square$

Example 3.6. *Take $f(z)=g(z)=h(z)=\frac{z}{1-z}$ as in Example 3.3, with $\alpha=\beta=\gamma=1$. Then,*

$$J_{1,1,1}(f,g,h)(z)=\frac{2}{z}\int_0^z\left(\frac{t}{(1-t)^3}e^{t/(1-t)}\right)dt. \tag{3.29}$$

One can verify directly that $\operatorname{Re}\left\{\frac{zJ'}{J}\right\}>0.6$ *for $|z|<0.5$, consistent with Theorem 3.2.*

4 Conclusion

We have introduced a new generalized integral operator that extends the operator studied by Eljamal and Darus (2013). Our operator incorporates multiple functions and additional parameters, providing greater flexibility in the study of univalent and starlike functions (Aldweby and Darus, 2025). We have established several mapping properties, including preservation of starlikeness, starlikeness of a given order, and relationships with Janowski classes. The results unify and extend many previously known results in the literature (Breaz et al., 2008; Mohammed et al., 2010; Mohammed and Darus, 2010; Breaz et al., 2010). Future research could explore further properties such as convexity, close-to-convexity, and applications to differential subordination.

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Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of manuscripts.

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Competing Interests

Authors have declared that no competing interests exist.

References

Aldweby, H. and Darus, M. (2025). Inclusion and closure properties for a generalized Janowski-q-starlike function class. *Communications of the Korean Mathematical Society*, 40(3):633–651. https://doi.org/10.4134/CKMS.c240202.

Bernardi, S. D. (1969). Convex and starlike univalent functions. *Transactions of the American Mathematical Society*, 135:429–446. https://doi.org/10.1090/S0002-9947-1969-0232920-2.

Breaz, D., Owa, S., and Breaz, N. (2008). A new integral univalent operator. *Acta Universitatis Apulensis*, 16:11–16. https://www.uab.ro/reviste$_old/auamath/docs/acta16/2008-1-2.pdf$.

Breaz, N., Breaz, D., and Darus, M. (2010). Convexity properties for some general integral operators on uniformly analytic functions classes. *Computers & Mathematics with Applications*, 60(12):3105–3107. https://doi.org/10.1016/j.camwa.2010.10.012.

Eljamal, E. A. and Darus, M. (2013). Inclusion properties for certain subclasses of p-valent functions associated with a generalized derivative operator. *Vladikavkaz Mathematical Journal*, 15(2):27–34. http://www.mathnet.ru/php/archive.phtml?jrnid=vmjwshow=contentsoption$_lang = eng$.

Janowski, W. (1973). Some extremal problems for certain families of analytic functions I. *Annales Polonici Mathematici*, 28:297–326. https://doi.org/10.4064/ap-28-3-297-326.

Miller, S. S. and Mocanu, P. T. (1981). Differential subordinations and univalent functions. *Michigan Mathematical Journal*, 28(2):157–171. https://cir.nii.ac.jp/crid/1573668923920946688.

Miller, S. S. and Mocanu, P. T. (2000). *Differential subordinations: Theory and applications*, volume 225 of *Pure and Applied Mathematics*. Marcel Dekker, New York. https://doi.org/10.1201/9781482289817.

Mohammed, A. and Darus, M. (2010). A new integral operator for meromorphic functions. *Acta Universitatis Apulensis*, 24:231–238. http://auajournal.uab.ro/.

Mohammed, A., Darus, M., and Breaz, D. (2010). Some properties for certain integral operators. *Acta Universitatis Apulensis*, 23:79–89. https://www.uab.ro/reviste/Acta/Acta/23.

Noor, K. I. (1987). On some univalent integral operators. *Journal of Mathematical Analysis and Applications*, 128(2):586–592. https://doi.org/10.1016/0022-247x(87)90208-3.

Pommerenke, C. (1975). *Univalent functions*. Vandenhoeck & Ruprecht, Göttingen. https://cir.nii.ac.jp/crid/1573668923920946688.

Reddy, G. L. and Padmanabhan, K. S. (1982). On analytic functions with reference to the Bernardi integral operator. *Bulletin of the Australian Mathematical Society*, 25:387–396. https://doi.org/10.1017/S0004972700005438.

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