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Modeling Dependence using Copula Garch

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

This study sought to investigate the tail dependence between government debt and bank's nonperforming loans. The objectives of this study were formulation of a bivariate copula model which captures the dependence between government debt and bank non-performing loans and measuring the tail and asymmetric dependence between the two variables, the study used quarterly data sourced from World Bank. To model the dependence between debt and bank non-performing, different methods have been used. The study estimated the dependence using copula GARCH, an approach that combines copula functions and GARCH models. According to forming the effect of local government debt and bank's non-performing loans, copula models have been applied to analyze the asymmetry of tail dependence structure between government debt exposure and bank non-performing loans. We used R programming language and Excel to plot and analyze data. The results showed that student t copula parameter provided the best fit for the marginal distributions. The results show the influence of government debt on bank non-performing loans. Further researchers should focus on time to ensure the effectiveness of risk measurement and management.

Keywords: Copula; tail dependence; Government debt; bank non-performing loans.

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1 Introduction

Financial crises frequently have a connection between government debt and non-performing loans from banks. In their recent history of financial crises, American economists [1] emphasized it. Over 60% of the crises related to government debt from 1970 to 2000 also involved financial problems.
Africa's problems with external debt is increasingly acknowledged to be a severe worldwide economic crisis. Numerous articles on the subject have been published in recent years, featuring research by [2], [3] , [4], [5], [6], and [7]. At the start of 1994, the Mexican bank loans opened a new cycle of crises financial. The latter heralds the first real crisis in which the weaknesses of the sector banking do not appear to be the ultimate consequence of a previous imbalance but as one of the first symptoms of a banking crisis [8]. Two years later, the latter was followed by the Thai crisis of 1997-98 which spread to much of Southeast Asia. This crisis was marked by devastating effects which quickly spread to all emerging countries where the banking system was destabilized and the currency crisis thus amplified.
The government debt crisis, and more precisely the euro zone crisis, seems to attest a strong interdependence between banking crises and debt crises government. On the one hand, a government debt crisis may have its roots in a financial crisis [9]. Indeed, it is the state that suffers most of the direct cost of the collapse of the banking system. This materializes as very high budgetary costs due to the provision of public funds to support the banking system. Because it affects financial institutions' financial statements, domestic banks' credit ratings, the value of their collateral, and their cost of borrowing, government debt has an impact on the financial industry.

Several studies have focused on showing the dependence between government debt and non-performing loans from banks. For instance,[10] , a struggling banking sector prompts government bailouts, whose cost raises the risk to the government's credit. In return, increased government credit risk affects the financial sector by depreciating the value of its holdings of bonds and government guarantees. According to [11] , banks with ratings profiles that are equivalent to or greater than those of their government states are more negatively impacted by government downgrades than banks with ratings profiles that are lower than their government states. According to [12], increasing policy uncertainty makes it harder for governments to give bailouts and links government risk to bank bailouts. The devastating effects of government default on overall financial activity in the defaulting country are the main emphasis of [13] research; the impact is worse in nations where domestic banks hold a larger share of the public debt. For a sample of euro-area banks, [14] point out the amplifying effect of governmental pressures on bank lending to domestic enterprises. All of the aforementioned evidence points to a significant feedback loop between public debt and non-performing bank loans.

The majority of financial research employ copula. Using copula-based models, [15] analyze several numerical techniques to aggregate various categories of risk (such as market risk and credit risk).

They examine Monte-Carlo simulation as well as a method of numerical integration and describe an approach for the calculation of the correlation between different risk types.Valdo D'Russo Toby Nikeghbali Copulas may be effectively utilized to address a variety of financial issues, as demonstrated by Gael Riboulet Thierry Roncalli in 2000. A common technique for approximating the joint distribution is the copula approach, and because of its accuracy, complexity, and adaptability to different assessment methods, many scholars are interested in studying it.

What makes copulas much more interesting is that they allow to separate the marginal distribution of the variables from their dependence, which considerably increases the flexibility in the specification and estimation of the model, they also make it possible to identify the level of dependency between variables as well as the kind of dependence, such as whether it is symmetrical or asymmetrical. Copulas are very general, encompassing a number of existing multivariate models and providing a framework for generating many more.

In this study, we describe the calculation of the tail dependence coefficients, which consisted of the copula functions, the model marginal distributions and the estimation of the parameters. For the investigation, we will use data from Burundi, Kenya, Rwanda, Republic Central Africa, Seychelles and Madagascar.

The relationship between bank non-performing loans and government debt can be modeled using families of bivariate distribution. Bivariate normal distribution is the most used because it is easy to apply and commonly studied, however it is not appropriate for the case where there is marginal distribution. Since the marginal distributions' models may be described independently of the dependence structure that ties these distributions together to produce a joint distribution, Copula's method solves the problem. As a result, the model's formulation and estimation are substantially more flexible. Using copula models, this work aimed to represent an asymmetric and tail dependency between the two variables.

The results of this study will have a significant impact on a range of people, including policymakers, banks, human rights organizations, and other scholars. Government decision-makers will undertake their research before deciding whether and how much domestic debt to issue to support government budgets in Burundi, Rwanda, Chad, Kenya, Madagascar, Republic Central Africa and Seychelles drawing from the results of this study. The findings of this study will also aid in developing stronger regulatory frameworks for other nations. The impact of public debt will also be communicated to commercial banks through assessment of the resulting bank balance. The findings will also be helpful to human rights organizations that care about the health of banks since they will enable them to determine if public debt is having a good or bad impact. As a result, their campaigns will have a strong foundation. Other researchers conducting related research will gain from this study's findings.

Consequently, their campaigns will have a solid base. Government and bank loan performance are important to an economy. The scope of this study is to show the relationship between government debt and bank nonperforming loans by using copula models. It shows how they can affect each other. Data will cover the period 2010-2020, which is considered as sufficient for analyzing the relationship.

2 Materials and Methods

2.1 Introduction

The approach used to simulate the correlation between the ratio of government debt to GDP and non-performing loans from bank is described in this chapter. The first aim, to design a copula GARCH model for bank non-performing loans and government debt, was addressed in the first section, which comprised a fundamental evaluation of the data, descriptive statistics, and the link between bank non-performing loans and government debt. In the second portion, many copula models were investigated to see how successfully the variables are connected.

2.2 Formulation of Bivariate Copula

2.2.1 Data description

Our data collection includes annual observations for the following developing nations: Burundi, Central Africa, Chad, Kenya, Madagascar, Rwanda, and Seychelles on government debt to GDP ratios and bank nonperforming loans. The sample spans the years 2010 through 2020 and consists of 15 columns and 11 rows. Using yearly statistics from seven Sub-Saharan African nations as a starting point, this section examines major developments in public debt in the area. Public debt for each of these nations refers to gross general government debt. Three stages in time are considered for evaluating public debt: in 2013, prior to the collapse in commodity prices in 2014, the year that signifies a turning point in debt dynamics, and in 2018, for which the most current statistics for the majority of the nations are available.

2.2.2 Normality test

There are a number of tests for normality, including the normal probability plot, the residuals histogram, and the Jacque-Bera tests. This study concentrated on the Jarque-Bera test since it takes skewness and kurtosis into account when analyzing data. Based on the data's skewness and kurtosis, a set of data values is subjected to the Jarque-Bera test to see if they match the normal distribution.

The The Jacque-Bera test is employed to determine check the fitted residuals for normality. It tests to see if the coefficients of excess kurtosis and skewness are both 0. The asymmetry of a probability distribution is assessed by skewness, and the degree of peakedness in reference to the tails is defined by kurtosis.

Hypothesis:

H0: The innovations (ϵ_t) are normally distributed

H1 : The innovations are not normally distributed.

The test statistic equation incorporating skewness and kurtosis is:

$$JB = N[\frac{S^2}{6} + \frac{(K-3)^2}{24}] \tag{2.1}$$

The number of values for each data point is n. The sample skewness, or S, measures how far the data deviates from the mean.

$$S = \frac{\frac{1}{n}\sum_{i=1}^{n}(x-\overline{x})^3}{(\frac{1}{n}\sum_{i=1}^{n}(x-\overline{x})^2)^{\frac{3}{2}}} \tag{2.2}$$

K is the sample kurtosis, which measures how thick the distribution's tails are.

$$K = \frac{\frac{1}{n}\sum_{i=1}^{n}(x-\overline{x})^4}{(\frac{1}{n}\sum_{i=1}^{n}(x-\overline{x})^2)^2} \tag{2.3}$$

The test statistic result will always be greater than or equal to zero since:

–The test statistic equation's sample skewness is always squared, hence S^2 is either always positive or 0.

–The sample kurtosis is always positive or zero since the numerator is raised to the 4th power and the denominator is squared.

–The difference between the sample kurtosis and 3 is squared, meaning this term of the test statistic equation is always positive or zero.

–The sum of two terms ≥ 0 be larger than or equal to zero as well.

When the p-value surpasses our 0.05 cutoff and the test statistic is near to zero, we may declare that the data is normally distributed. The p-value is linked to the null hypothesis, this asserts that the data is regularly distributed. The data is not normally distributed, When the test statistic is significant and the p-value is less than 0.05. The null hypothesis is disregarded if the p-value is less than 0.05 since the statistic has a $\chi 2$ distribution.

2.2.3 Measures of correlation

Kendall's Tau Coefficient Maurice Kendall proposed the Kendall's tau coefficient was developed in 1938 is a statistical measure of the nonlinear relationship among the random variables X and Y. The connection between the two variables is monotonic, albeit not always linear, according to Kendall's tau.

A nonparametric hypothesis test based on the tau correlation coefficient is called Kendall's tau test. If X_iX_j and Y_iY_j or $X_i > X_j$ and $Y_i > Y_j$, When the pairings to each variable are in the same order, the points (X_j, Y_j) and (X_i, Y_i) are said to be concordant. In other words, a higher X value creates a higher Y value, and a lower X value causes a lower Y value. The values of the variables are organized in opposing orientations, with a lower X value corresponding to a higher Y value and a higher X value corresponding to a lower Y value, and these pairings are said to be discordant if X_iX_j and $Y_i > Y_j$ or $X_i > X_j$ and Y_iY_j. The pairings are linked if $X_i = X_j$ and/or $Y_i = Y_j$. If two random variables X and Y follow the joint distribution $H(x, y)$ and if two vectors of

(X_i, Y_i) and (X_j, Y_j) are independent, then Kendall's tau is defined:

$$\tau = P(Xi–Xj)(Yi–Yj) > 0–P(Xi–Xj)(Yi–Yj) < 0 \tag{2.4}$$

The result calculated by this equation falls inside the range $[-1, 1]$, matching Pearson's correlation. The correlation between the two variables is stronger the larger the absolute value of. A positive value, on the other hand, indicates the inverse, that is, the higher value of one variable has no bearing on the higher value of another.

Spearman's Rho To understand Spearman's correlation, you must first understand what is a monotonic function. A monotonic function is one that never rises or never falls as the independent variable rises. When the x variable rises but the y variable never falls, rises but the x variable never falls, and rises but the y variable periodically falls and occasionally rises, the system is said to be non-monotonic growing. The strength of a monotonic relationship between matched data is gauged by the Spearman's correlation coefficient. It appears as r_s in a sample and is constrained by design in the manner that is described below.

$$-1 <_= r_s <_= 1$$

Pearson linear correlation coefficient The Spearman's correlation coefficient is a statistical measure of the strength of a monotonic relationship between paired data. It is confined by design in the way explained below and appears in a sample as r_s.

$$\rho(X, Y) = \frac{cov(X, Y)}{\sigma X \sigma Y}$$

One of its extensions is the Spearman's rank correlation coefficient. These techniques are easy to use and are supported by the majority of statistical software. Nevertheless, they are restrained. For example, they can only detect linear or monotonic correlations and can only handle two univariate random variables. These traditional approaches fall short in addressing the novel problems created by huge data volumes. Big data objects may actually be anything from high-dimensional vectors to real-time feeds, images, and texts. The linear relationship is shown using the Pearson coefficient. A linear connection is the most significant one between random variables. When there is a linear relationship between two sets of continuous data, those data have a general normal distribution, and each set of measured values is independent of the other, the Pearson correlation coefficient is helpful.

The following is the formula.

$$P_{x,y} = \frac{\sum xy - \frac{\sum_x \sum_y}{n}}{\sqrt{(\sum x^2 - \frac{(\sum x)^2}{n})(\sum y^2 - \frac{(\sum y)^2}{n})}} \tag{2.5}$$

where the variables are x and y and n is the number of values. The value range can be used to evaluate the relative efficacy of components under normal circumstances.

2.2.4 Formulation of Bivariate

Before we can evaluate the correlation between inflation and exchange rate, the data must be translated into log returns. Let

$$R_t = \log(\frac{X_t}{X_{t-1}}) \tag{2.6}$$

$$P_t = \log(\frac{Y_t}{Y_{t-1}}) \tag{2.7}$$

where X_t is inflation and R_t is the log returns of inflation. Y_t represents exchange rate and P_t is the exchange rate log returns.

Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Bollerslev (1986) and Taylor (1986) presented an expansion of the ARCH model. According to the model, u's conditional variance at time t is determined by the error term squared and its conditional variance in earlier periods.The following is the definition of the GARCH(1,1) model:

$$\sigma_t^2 = \beta_0 + \beta_1 \mu_{t-1}^2 + \alpha_1 \sigma_{t-1}^2 \tag{2.8}$$

where $\beta_0 \geq 0$,$\beta_1 > 0$ for $i = 1, ...q$ and $_i$ is $\geq$ for $i = 1, ...,$ β_0 determines the sensitivity of volatile conditions to market disturbances, β_1 is a metric for the persistence of conditional volatility, and if β_1 is strong, The volatility is going to last for quite a while. The relationship between α_1 and β_1 influences the rate at which conditional volatility converges for a long time unconditional volatility. The unconditional variance of μ_t is provided.

$$Var(\mu_t) = \alpha_0[1 - (\alpha_0 + \beta_0]^-1 \tag{2.9}$$

where stationarity in variance is implied by $\alpha_1 + \beta_1$ 1. As the horizons lengthen, the conditional variance forecast won't converge on an unconditional value, if $\alpha_1 + \beta_1 > 1$, which indicates nonstationarity in variance. The model may be enlarged to GARCH(q,p), where q lagged terms are square error terms and p is a conditional variance term.;

$$\sigma_t^2 = \beta_0 + \sum_{i=1}^{q} \beta_i \mu_{t-i}^2 + \sum_{j=1}^{p} \alpha_j \sigma_{t-j}^2 \tag{2.10}$$

To circumvent an ARCH model limitation, the GARCH model is employed. It saves money and avoids overfitting. It also pays less attention to breaking prohibitions against negativity (Cuthbertson, 2004). In this research, we will focus on the regular GARCH model, the exponential GARCH model, and the GJRGARCH model. The GARCH model has experienced several revisions and expansions.

(i) GARCH Standard Model

sGARCH represents the standard GARCH (q,p) model:

$$\sigma_t^2 = \omega + \sum_{i=1}^{m} \phi_i \mu_{it} + \sum_{j=1}^{q} \beta_i \mu_{t-1}^2 + \sum_{j=1}^{q} \alpha \sigma_{tj}^2 \tag{2.11}$$

where ω represents the constant term and μ_{it} represents exogenous variables.

(ii) GARCH exponential model

$$log(\sigma_t^2) = \beta_0 + \alpha \log(\sigma_{t-j}^2) + \gamma \frac{\mu_{t-1}}{\sqrt{\sigma_{tj}}} + \beta_1 [\frac{|\mu_{t-1}|}{\sigma_{t-j}} - \sqrt{\frac{2}{\pi}}] \tag{2.12}$$

In comparison to pure GARCH specifications, the model provides benefits. Because the $\log(\sigma_t 2)$ is modeled, σ_{t-j} will be positive even if the values of the parameters are negative. Because *gamma* is negative if the correlation between volatility and return is negative, the eGARCH formulation allows for asymmetries. Conditionally normal errors are employed for mistakes in eGARCH models rather than the generalized error distribution (GED) structure because they are easier to compute and understand intuitively.

(iv) The GARCH-GJR model The leverage impact was added by inserting a factor in the volatility formula to explain asymmetric volatility clustering. The GJR-GARCH model, commonly known as the Threshold GARCH model, is written as

$$\sigma^2 = \beta_0 + \sum_{j=1}^{q} \beta_i \mu_{t-1}^2 + \sum_{j=1}^{P} \alpha_j \sigma_t \,_{-j}^2 + \gamma \mu_{t-1}^2 \tag{2.13}$$

$I_{t_1=1}$ assuming and just assuming ϵ_{t-i}¡0 representing the negative indicator variable ϵ_{t-i} where $\epsilon_{t-i} = 1$ if ϵ_{t-i}¡0 and $\epsilon_{t-i} = 0$ otherwise.

Copula GARCH models, according to [16] Jondeau and Rockinger (2006), are a form of model in which some copula parameters may change temporally in an auto-regressive way conditioned on historical data. Individual return variation can be expressed by fitting an ARGARCH model. Because financial return distributions are often non-elliptical, the conditional approach is a crucial tool for minimizing heteroscedasticity. Copula parameters can change over time when using a time-varying copula, which aids understanding of dependent structure.

Model diagnostics It must be determined the way the model accommodates fluctuations with regard to the dependent variables. The goodness of fit test is employed to evaluate the model's accuracy and the significance of the parameters. The goodness of fit test, for example, will be put to use assess the applicability of the GARCH model for computing volatility. Several statistical tests will be utilized in this study to evaluate the fit of a GARCH(1,1) model.

2.3 Measure of Tail Dependence

2.3.1 Copula Models

The Sklar theorem introduced copulas, and Joe (1997) further developed them. A copula is a function which connects a one-dimensional marginal multivariate distribution of related variables to a univariate distribution of variables.

$$H(x,y) = C(FX(x), FY(y)) \tag{2.14}$$

Where $F_X(.)$ and $F_Y(.)$ are X's and Y's accumulative distribution functions and $H(.)$ is a bivariate function. According to Sklar's theorem, the dependent structure may be represented by a copula, hich is C, and the univariate margins can be separatd from it. Function $C : [0,1]^2 ß[0,1]$ is known as a bivariate copula.With some characteristics, which are $u = F_X(x)$ and $v = F_Y(y)$, are as follows:

– The domain of $C(u,v) is [0,1]x[0,1]$ – $C(u,1) = C(1,u) = u, C(v,1) = C(1,v) = v, u, v[0,1]$ – $C(u,v)$ has zero fundamentals and is incremented in two dimensions

Copulas are an effective technique to model dependent random variables, as the preceding definition shows. In dependence studies, copulas have a few specific qualities that are highly helpful. Even when the random variable transformation is considerably enhanced, copulas remain constant.Additionally, copulas have characteristics that check the consistency of frequently used random variables like Kendall's tau and Spearman's rho. The third characteristic of copulas is known as tail dependency.

The upper and lower tail dependency coefficients for are defined by Nelsen: $\tau^U[0,1]$ and $\tau^L[0,1]$ of (X,Y).

$$\tau^U = \lim P[U > \epsilon | V > \epsilon] = \lim P[V > \epsilon | U > \epsilon] \tag{2.15}$$

$$\tau^L = \lim P[U > \epsilon | V > \epsilon] = \lim P[V > \epsilon | U > \epsilon] \tag{2.16}$$

The upper (right) and lower (left) tail dependencies of a normal copula are equal, or $\tau_L = \tau_U = 0$, indicating that variables are independent at the extreme of the distribution. Even though it lacks tail dependence, the normal copula is the most often accepted assumption in finance. Copulas rely on either the left or right tail in different ways.

2.3.2 Tail dependence

The following are the definitions of "lower tail dependence" and "upper tail dependence": Assume that X_1 and X_2 represent two random variables with CDFs of F_1 and F_2. The upper tail dependence coefficient, λ, is then calculated as follows:

$$\lambda_u = \lim_{t \to 1^-} P[G(X) > t | H(Y) > t]$$

If the limit exists, the upper tail-dependence coefficient (upper TDC) is used. If $\lambda_u > 0$, then (X, Y) is upper tail dependent; otherwise, it is upper tail independent. Similarly, the lower tail-dependence coefficient is defined as

$$\lambda_l = \lim_{t \to 0^+} P[G(X) \geq t | H(Y) \geq t] \tag{2.17}$$

3 Results

3.1 Introduction

Results for the two objectives are provided in this chapter. Bivariate copula modeling is covered in part one, while quantifying tail dependency is covered in section two.

3.2 Formulate bivariate copula

3.2.1 Data exploration

Each of the sample nations' government debt to GDP ratios are displayed in Figure 1 for comparison. With the exception of Seychelles, whose examination began in 2011, all nations' changes in these percentages between 2012 and 2020 are quite comparable. Numerous example nations (with developed and developing economies) now face a major issue with high levels of public debt.

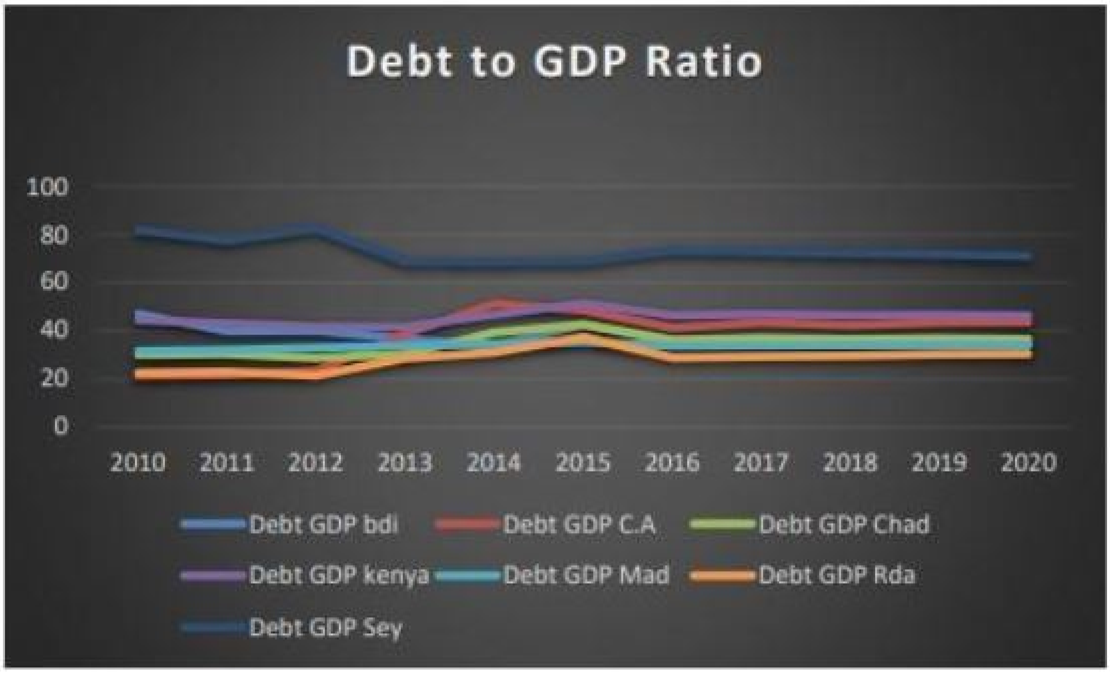

Fig. 1. Government debt to gross domestic product (GDP) ratio of the sample countries since during 2010-2020.Source: The World Bank, Data

Fig. 2 shows that, with the exception of Madagascar, which runs from 2010 to 2016, the sample nations' bank non-performing loan ratios varied significantly between 2012 and 2020.

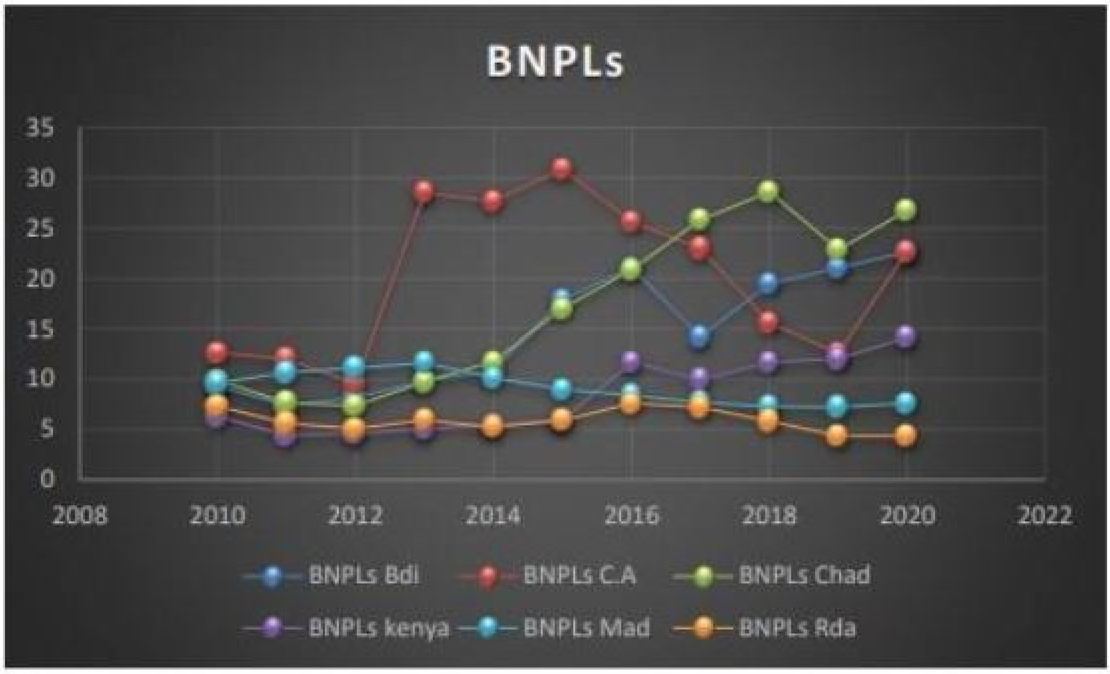

Fig. 2. Bank non-performing loans (NPLs) ratio of the sample countries since during 2010–2020.Source: The World Bank, Data

3.2.2 Descriptive Statistic

A brief overview The descriptive statistics in Table 1 are divided into two panels: Panel A represents the ratio of debt to GDP, while Panel B represents the ratio of non-performing bank loans.

The mean for the country's government debt to GDP and bank non-performing loan ratios is far from zero. As can be observed, several countries' averages are higher than those of other countries. The important variables exhibit significant time series volatility, as seen by the minimum/maximum and standard deviations. For instance, the ratio of government debt to GDP in Rwanda peaked at 9.22 basis points (panel A), whereas the ratio of non-performing bank loans in Madagascar peaked at 35.53 basis points (panel B). Burundi's bank non-performing loan ratio and Rwanda's government debt to GDP ratio both hit record highs during this time, at 7.62 and 46.91 basis points, respectively.

Nonetheless, the mean values of the key variables are typically fairly close to the average values. As observed in panel B, the Seychelles has larger negative values for skewness than the other countries, indicating a higher possibility of significant reductions and distributions with long left tails. Furthermore, Burundi, Chad, Kenya, Rwanda, and Madagascar all exhibit lengthy right tails, indicating positive skewness. Given that a normal distribution's kurtosis is typically 3, panel B demonstrates that the non-performing loan ratio for the bank has a kurtosis that is less than 3, indicating that the distribution is flat and has thin tails. For Central Africa and Madagascar, negative values for skewness are more pronounced in panel A, indicating that those distributions have extended left tails.

Additionally, there is proof of positive skewness for Kenya, Seychelles, and Burundi, which suggests that these countries have extended right tail distributions. Panel B demonstrates that the government-to-GDP ratio's kurtosis distribution is flat with short tails.

Table 1. Descriptive Statistic

Countries	Mean	Med	Min	Max	St.D	Skewness	Kurtosis
Panel A:Debt to GDP ratio							
Burundi	37.49	35.57	33.87	46.91	4.27	0.94	-0.50
Central African Republic	38.04	42.21	21.43	51.15	10.69	-0.59	-1.37
Chad	34.95	36.47	28.78	42.61	4.39	0.02	-1.37
Kenya	45.52	46.34	41.49	51.33	2.78	0.27	-0.49
Madagascar	33.79	34.03	31.69	35.53	1.10	-0.51	-0.74
Rwanda	28.47	29.45	21.46	37.29	4.57	0.01	-0.78
Seychelles	73.55	72.22	68.63	82.54	4.94	0.76	-0.99
Panel B:Bank NPLs							
Burundi	14.71	14.19	7.39	22.61	5.83	0.06	-1.89
Central African Republic	20.06	22.66	9.58	30.86	7.73	-0.01	-1.83
Chad	17.13	17.01	7.37	28.63	8.17	0.10	-1.84
Kenya	8.30	6.29	4.43	14.14	3.61	0.28	-1.79
Madagascar	9.12	9.01	7.21	11.63	1.61	0.19	-1.69
Rwanda	5.86	5.81	4.37	7.62	1.12	0.24	-1.40
Seychelles	6.85	7.59	3.49	9.22	1.93	-0.38	-1.47

3.2.3 Normality test

Because Jarque-Bera statistics are not very important, Tables 2 and 3 show that all of the indices are normally distributed. The null hypothesis that the data in the situation are normally distributed cannot be disproved if the p-value is higher than 0.05.

Table 2. Jarque Bera Statistics (Government debt for GDP ratio)

countries	x-squared	df	p-value
Burundi	2.1358	2	0.3437
Central African Republic	1.3397	2	0.5118
Chad	0.48547	2	0.7845
Kenya	0.17312	2	0.9171
Madagascar	0.67534	2	0.7134
Rwanda	0.045844	2	0.9773
Seychelles	1.5776	2	0.4544

Table 3. Jarque Bera Statistics (Bank's NPLs for GDP ratio)

countries	x-squared	df	p-value
Burundi	1.2732	2	0.5291
Central African Republic	1.1504	2	0.5626
Chad	1.1994	2	0.549
Kenya	1.2753	2	0.5285
Madagascar	1.0018	2	0.606
Rwanda	0.66504	2	0.7171
Seychelles	0.95875	2	0.6192

3.2.4 Correlation analysis

The correlation between each pair of sample countries is displayed by Kendall's tau. Calculated between each nation and the other nations is Kendall's tau. The means of Kendall's tau between the government-to-GDP ratio and bank non-performing loans are shown in Table 2 for the study countries.

The greatest levels of debt are found in Central Africa—Madagascar, Madagascar, and Rwanda, for instance—compared to other countries. The countries with the lowest debt levels include Rwanda, Burundi, Central Africa, and the Seychelles. The means between Chad and Madagascar and Kenya and Madagascar are the lowest when it comes to the mean of non-performing bank loans, followed by those between Burundi and Chad, Burundi and Kenya, and Madagascar and Seychelles.

For example, whereas the connection between Chad and Seychelles is negative, the correlation between Chad and Kenya is positive. The variation of relationships between nations may be due to differences in national characteristics. Since strong domestic sovereignty has a negative knock-on effect on bank risk, maintaining healthy public finances is necessary to shield the banking sector from its repercussions.

Table 4. Kendall's tau mean

countries	Mean of Debt	Mean of Bank NPLs
Burundi-Central Africa	-0.6727	0.2
Burundi-Chad	-0.4909	0.6727
Burundi-Kenya	-0.2727	0.7818
Burundi-Madagascar	-0.5636	-0.6
Burundi-Rwanda	-0.6	-0.1272
Burundi-Seychelles	0.4909	-0.5272
Central Africa-Chad	0.7454	0.0909
Central Africa-Kenya	0.5272	-0.0181
Central Africa-Madagascar	0.8909	0.1272
Central Africa-Rwanda	0.8545	0.2363
Central Africa-Seychelles	-0.6727	0.1272
Chad-Kenya	0.7454	0.7454
Chad-Madagascar	0.7818	-0.7818
Chad-Rwanda	0.8181	-0.0181
Chad-Seychelles	-0.5636	-0.5636
Kenya-Madagascar	0.5636	-0.7454
Kenya-Rwanda	0.6	-0.0545
Kenya-Seychelles	-0.3454	-0.6727
Madagascar-Rwanda	0.8909	0.0909
Madagascar-Seychelles	-0.6363	0.7818
Rwanda-Seychelles	-0.7454	0.0909

3.2.5 Formulation of bivariate copula

A GARCH (1,1) model was fitted for each variable across all nations to account for volatility. AIC and BIC were used to compare the different GARCH model parameters for each country's government debt and bank non-performing loans. The selected model has a lower AIC or BIC.

Tables 5 and 6 exhibit AIC summary for bank non-performing loans and government debt. The table shows why the gjrGARCH volatility model was chosen for Burundi returns. The eGARCH model is employed in Central Africa, Chad, Kenya, Madagascar, Rwanda, and Seychelles. The proper GARCH model parameters were then used to fit each return for bank non-performing loans and government debt.

Table 5. AIC for debt GDP GARCH

countries	sGARCH	eGARCH	gjrGARCH	Min
Burundi	-1.9484	-1.8494	-2.0000	2.2e-16
Central African Republic	0.352255	-1.8728	-0.280804	2.2e-16
Chad	-0.84412	-3.9367	-1.8756	2.2e-16
Kenya	-1.9869	-3.8326	-3.0687	2.2e-16
Madagascar	-3.9386	-7.6154	-5.6010	2.2e-16
Rwanda	-0.168216	-2.9616	-2.5204	2.2e-16
Seychelles	-2.1550	-4.9353	-2.2183	2.2e-16

Table 6. AIC for Bank NPLs GARCH

countries	sGARCH	eGARCH	gjrGARCH	Min
Burundi	-0.141214	-0.309344	-1.05524	2.2e-16
Central African Republic	2.7812	-0.83405	2.9053	2.2e-16
Chad	0.47568	-0.68514	-0.31639	2.2e-16
Kenya	1.2844	-1.9360	0.232722	2.2e-16
Madagascar	-1.5471	-2.7801	-2.7973	2.2e-16
Rwanda	-0.47506	-2.9616	-2.3388	2.2e-16
Seychelles	1.28599	-0.52269	1.35902	2.2e-16

The metric $\beta 1$ from Tables 7 and 8 measures the persistence in conditional volatility for the government debt and bank non-performing loans regardless of what is happening in the nation. The $\beta_1 s$ are all positive, considerably less than 0.9, with the exception of Burundi, Chad, Madagascar, Rwanda, and Seychelles for government debt. They are also significantly above 0.9, with Burundi being the exception for bank non-performing loans, which indicates that volatility takes a while to decay. Tables 9 and 10 include Ljung box (R) test data for debt to GDP. The BNPLs fitted residuals are independently distributed since the p value is less than 0.05 and there is high auto correlation at different. For and the Ljung box (R^2) are substantially larger than 0.05. Because of this, we are unable to disprove the test's null hypothesis and draw the inference that the data values are independent.

3.3 Measure of tail dependence

3.3.1 Copula results

Three copula models are used to examine the tail dependency between the government debt to GDP ratio and the bank non-performing loan ratio. Joe, Clayton, and Student's t copulas were rotated, correspondingly, for the symmetric and higher tail dependent connections. These series copula models, which are the most often used copulas in finance, encompass the important feature combinations needed to capture possible correlations between the variables under consideration.

Copula families representing marginal normalized to uniform distributions are drawn from both parametric (such as the Rotated Clayton Copula and Joe Copula) and non-parametric (such as the Student's T Copula) families. We take into account the pairings of each nation's government debt and bank non-performing loans while analyzing the copula model's parameters. Table 9 shows an estimate of the results from the three copula models. The student t copula is used to determine the upper and lower tail dependencies for each sample nation. Due to the symmetry of the student's t distribution, the larger tail coefficient frequently matches the lower tail coefficient. We observe that Chad has the largest upper tail dependence of about 0.725. The significant rise in bank non-performing loans and the disproportionate growth of government debt are clearly related, as shown by the strong and positive tail dependency.

Table 7. Estimates of the GARCH (1,1) parameters under various specifications for each of government debt

countries	Parameters	Estimate	Std. Error	t value	Pr($\succ |t|$)
5*Burundi	μ	-0.013643	0.001213	-11.25074	0.000000
	α	0.000020	0.000017	1.15333	0.248776
	α_1	0.033494	0.050463	0.66374	0.506855
	β_1	0.000000	0.036190	0.00000	1.000000
	γ	1.000000	0.470428	2.12572	0.033526
5*C.A	μ	-0.058472	0.000034	-1705.69	0
	α	0.343977	0.000465	739.28	0
	α_1	-2.234489	0.001151	-1941.77	0
	β_1	0.990000	0.000511	1938.58	0
	γ	-4.833421	0.002328	-2076.35	0
5*Chad	μ	-0.004826	0.000004	2061.3	0
	α	-2.213717	0.002406	-3144.7	0
	α_1	3.934497	0.002180	2778.6	0
	β_1	0.989833	0.000111	2241.0	0
	γ	-8.405392	0.001588	-5460.4	0
5*Kenya	μ	-0.004826	0.000011	-431.40	0
	α	-2.213717	0.002825	-783.55	0
	α_1	3.934497	0.000683	5757.42	0
	β_1	0.989833	0.002093	472.85	0
	γ	-8.405392	0.007109	-1182.32	0
5*Madagascar	μ	0.004926	0.000008	587.87	0
	α	-8.948875	0.010919	-819.55	0
	α_1	6.578835	0.033142	198.50	0
	β_1	0.226748	0.000025	9127.84	0
	γ	-3.252094	0.009990	-325.54	0
5*Rwanda	μ	0.008737	0.000005	1725.1	0
	α	-0.045188	0.000028	-1639.6	0
	α_1	2.180732	0.000968	2252.8	0
	β_1	0.369815	0.000255	1451.3	0
	γ	-4.580190	0.003403	-1346.0	0
5*Seychelles	μ	-0.00658	0.000003	-2263.208	0
	α	-0.84670	0.002059	-411.287	0
	α_1	2.18272	0.042224	51.694	0
	β_1	0.98964	0.000164	6032.986	0
	γ	7.70155	0.314725	24.471	0

The Student t distribution, as well as the rotated Clayton and Joe copula distributions, stress more tail dependency as opposed to less tail dependence. We looked examined the top tail dependency between government debt and bank non-performing loans using the Clayton and Joe Copula alternative methods. We see that two out of seven countries, whether using the rotating Clayton copula or the Joe copula, both of which are symmetric copula functions, have an upper tail dependency value smaller than 0.5.

3.3.2 conclusion

Through its investigation of the tail connection between government debt and bank non-performing loans, this study adds to the body of financial literature. The study's findings support the assertion that, with the exception

Table 8. Estimates of the GARCH (1, 1) parameters under various specifications for each of bank non-performing loans

countries	Parameters	Estimate	Std. Error	t value	Pr($\succ \|t\|$)
	μ	-0.051997	0.000056	-936.35	0
	α	-2.012603	0.002113	-952.48	0
5*Burundi	α_1	0.037984	0.000169	225.18	0
	β_1	0.996603	0.002617	380.78	0
	γ	-0.284079	0.000761	-373.23	0
	μ	-0.17023	0.000076	-2238.3	0
	α	-3.51931	0.001653	-2129.3	0
5*C.A	α_1	-2.69796	0.001125	-2398.5	0
	β_1	0.20049	0.000046	4397.5	0
	γ	-7.55273	0.003518	-2147.1	0
	μ	0.20022	0.000089	2244.9	0
	α	-1.32825	0.000734	-1809.4	0
5*Chad	α_1	-2.13342	0.000357	-5980.1	0
	β_1	0.27064	0.000103	2625.3	0
	γ	-5.62261	0.002706	-2078.0	0
	μ	0.098425	0.000041	2385.3	0
	α	-2.816527	0.001947	-1446.9	0
5*Kenya	α_1	1.660220	0.000311	5336.7	0
	β_1	0.313736	0.000071	4416.9	0
	γ	-5.249622	0.003552	-1478.0	0
	μ	-0.025048	0.000006	-4391.3	0
	α	0.570912	0.000446	1280.5	0
5*Madagascar	α_1	1.581390	0.000932	1697.2	0
	β_1	0.270490	0.000127	2134.3	0
	γ	-5.501157	0.003606	-1525.5	0
	μ	0.008737	0.000005	1725.1	0
	α	-0.045188	0.000028	-1639.6	0
5*Rwanda	α_1	2.180732	0.000968	2252.8	0
	β_1	0.369815	0.000255	1451.3	0
	γ	-4.580190	0.003403	-1346.0	0
	μ	0.021758	0.000007	3031.2	0
	α	0.174662	0.000102	1711.7	0
5*Seychelles	α_1	-1.779849	0.001030	-1727.9	0
	β_1	0.497674	0.000271	1839.1	0
	γ	-5.748830	0.003465	-1659.3	0

of Seychelles, the ratio of government debt to non-performing loans in every other nation climbed considerably between 2012 and 2020. These data may be used by financial institutions to evaluate which countries represent the biggest threat to regional stability in the event of a final default.

4 Discussion, Conclusion and Recommendation

4.1 Introduction

This chapter gives the discussion, conclusion and the recommendation of the study.

Table 9. Ljung-Box Test on Standardized and Residuals Squared for debt GDP

countries	ljung-Box (R)	p.value	ljung-Box (R^2)	p.value
Burundi	4.834	2.2e-16	3.3841	0.06583
Central African Republic	6.6634	2.2e-16	5.1257	0.02357
Chad	1.0175	2.2e-16	0.712	0.3987
Kenya	0.00370	2.2e-16	0.00277	0.958
Madagascar	4.7096	2.2e-16	3.6228	0.05699
Rwanda	1.8924	2.2e-16	1.4557	0.2276
Seychelles	1.544	2.2e-16	1.188	0.2757

Table 10. Ljung-Box Test on Standardized and Residuals Squared for BNPLs

countries	ljung-Box (R)	p.value	ljung-Box (R^2)	p.value
Burundi	3.641	2.2e-16	2.5489	0.1104
Central African Republic	2.5074	2.2e-16	1.9288	0.1649
Chad	0.586	2.2e-16	0.439	0.5073
Kenya	4.5398	2.2e-16	3.3016	0.06921
Madagascar	2.3888	2.2e-16	1.7916	0.1807
Rwanda	1.4777	2.2e-16	1.0344	0.3091
Seychelles	0.02251	2.2e-16	0.016883	0.8966

Table 11. Tail dependence copula between government debt and bank non-performing loans

	tCopula		clayton copula		joe copula	
countries	lower	upper	lower	upper	lower	upper
Burundi	0.37816554	0.37816554	0	0.897464032	0	0.805283369
C. A	0.30943879	0.30943879	0	0.350676814	0	0.152003691
Chad	0.72549674	0.72549674	0	0.854655378	0	0.968894786
Kenya	0.33726254	0.33726254	0	0.516187222	0	0.505163091
Madagascar	0.10711712	0.10711712	0	0.657799910	0	0.562899169
Rwanda	0.02737502	0.02737502	0	0.405290549	0	0.296952087
Seychelles	0.17950360	0.17950360	0	0.765840416	0	0.864908886

4.2 Discussion

The purpose of this study was to model the link between government debt and bank non-performing loans using the copula. We received quarterly figures from the World Bank. The data was collected between 2010 and 2020. We concluded that the Studet t copula was the best for government debt and bank non-performing loans. It was revealed that the GARCH model (1,1) best represented volatility for each country. The results show a dependence relationship between the two factors.

Unlike previous research, this one focused on developing economies and studied the growing body of literature on copula-based time series models for the financial and economic sectors. The marginal distributions and copula are commonly declared to belong to parametric families when these models are calculated semiparametrically or parametrically. Using copula models, a similar problem has been identified for developed nations.

4.3 Conclusion

A copula model is used in this study to describe the relationship between government debt and bank non-performing loans. The World Bank provided us with R-annotated annual data. The data spans the years 2010 through 2020. We begin by constructing stationary series for non-performing bank loans and government debt. Kendall's tau was used as a replacement indicator to assess the association between government debt and bank non-performing loans in each pair of countries. Madagascar and Central Africa have considerably higher internal correlations of government debt than other areas, we observed. The greatest correlation means of bank non-performing loans are seen in Chad and Madagascar.

The bivariate Copula is subsequently constructed. The GARCH model (1,1) was fitted to each nation to capture volatility. To compare the various GARCH model parameters, AIC and BIC were employed. It was chosen to go with a model that had a lower AIC and BIC.

Three copula functions were employed to investigate the upper tail dependence between government debt and bank non-performing loans. We identified a significant disparity in tail reliance between countries.

According to the considerable and positive tail dependence, the unprecedented increase of government debt and the rapid rise in non-performing bank loans are deemed to be significantly associated. These countries' higher tail dependence coefficients imply that bank non-performing loans were more vulnerable to the growth in public debt over our sample period.

4.4 Recommendation

This research will assist a variety of institutions. These findings will help some institutions set up a control mechanism that will aid in rebalancing excessive government debt and understanding the relationship between government debt and bank non-performing loans.

Future research will benefit from more broad data in order to comprehend the advantages more clearly of the copula function for various financial occurrences.The research can potentially be extended by incorporating more macroeconomic elements over a longer time period.

Competing Interests

Authors have declared that no competing interests exist.

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