MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2017/2018 ACADEMIC YEAR
FOURTH YEAR SECOND SEMESTER

SCHOOL OF SCIENCE
BACHELOR OF SCIENCE IN APPLIED STATISTICS WITH COMPUTING

COURSE CODE: STA 424
COURSE TITLE: STOCHASTIC PROCESSES

DATE: 15TH APRIL 2019 **TIME: 8.30AM -10.30AM**

INSTRUCTIONS TO CANDIDATES

1. Answer Question **ONE** and any other **TWO** questions.
2. Show all your Workings.

QUESTION 1

a). An analyst was contracted by a potential investor in mobile network services targeting the mobile money transfer services to investigate on the behavior of user's frequency of switching from one network to another. The analyst realized that the three major operators were Safaricom, Airtel and Orange. Using this information on how frequently users tends to switch social network with respect to time describe;

i). A stochastic process. **[2 Mark]**

ii). Parameter space. **[2 Mark]**

iii). State space. **[2 Mark]**

b). At what average rate must a cashier at a the university cafeteria work in order to ensure a probability of 0.90 that a customer will not wait longer than 12 minutes. Assume only that one counter, customers arrive in a Poisson fashion at average rate of 15/hr and length of service by the cashier has exponential distribution. **[4 Marks]**

c). The following Markov chain with five states describes transition in a busy banking facility of customers seeking different services, the states are E_0, E_1, E_2, E_3 and E_4 and the transition probabilities

$$p_{0,2} = p_{4,2} = 1$$

$$p_{1,2} = p_{2,3} = p_{3,4} = \frac{1}{4}$$

$$p_{1,0} = p_{2,1} = p_{3,2} = \frac{3}{4}$$

$$p_{i,j} = 0 \ \ otherwise$$

i). Find the stochastic matrix. **[3 Marks]**

ii). Prove that the Markov chain is irreducible. **[4 Marks]**

iii). Find the invariant probability vector. **[4 Marks]**

d). Define the following terms as used in stochastic processes.

i). Transient State. **[1 Marks]**

ii). Mean Recurrence time. **[2 Marks]**

e). i). Define a Queuing model. **[2 Mark]**

ii). Describe the four characteristics of a queuing system **[4 Marks]**

QUESTION 2

a). Define the following terms as use in stochastic processes.

i). Markov chain. **[3 Marks]**

ii). irreducible Markov chain. **[2 Marks]**

iii). invariant probability vector. **[2 Marks]**

b). Given a Markov chain of five states E_1, E_2, E_3, E_4 and E_5, and transition probabilities;

$$p_{i,i+1} = \frac{i}{i+2},\ p_{i,1} = \frac{2}{i+2};\ i = 1, 2, 3, 4$$

$$p_{5,1} = 1, \qquad p_{i,j} = 0\ otherwise$$

i). Find the stochastic matrix. **[2 Marks]**

ii). Prove the Markov chain is irreducible. **[1 Mark]**

iii). Prove the Markov chain is regular. **[1 Mark]**

iv). Find the invariant probability vector). **[3 Marks]**

v). Assume that the process at time $t = 0$ is in state E_1. Denote by T the random variable, which indicates the time of first return to E_1. Find $P\{\ T = k\}, k =\ 1, 2, 3, 4, 5$, and compute the mean and variance of T **[6 Marks]**

QUESTION 3

a). A certain piece of equipment is inspected at the end of each day and classified as just overhauled, good, fair or inoperative. Let us denote the four classifications as states 1, 2, 3 and 4 respectively. If the item is inoperative it is overhauled, a procedure that takes one day. assume that the working condition of the equipment follows a Markov process with the following state transition matrix

$$\begin{array}{c} \text{Tomorrow} \\ \text{today} \begin{pmatrix} 0 & 3/4 & 1/4 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 1 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

It costs Kshs. 125 000 to overhaul machine on the average and Kshs. 75 000 is lost in production, if a machine is found inoperative. Use the steady state to compute the expected per day cost of maintenance. **[6 Marks]**

b). Five balls, 2 white ones and 3 black ones are distributed in two boxes A and B, such that A contains 2 and B contains 3 balls. At time n (where $n = 0, 1, 2, \ldots$) we choose at random from each of the two boxes one ball and let the two chosen balls change boxes. In this way we get a Markov chain with 3 states $E_0, E_1,$ and E_2, according to whether A contains $0, 1$, or 2 black balls.

i). Find the corresponding stochastic matrix. **[3 Marks]**

ii). Prove that the Markov chain is regular. **[2 Marks]**

iii). Find the invariant probability Vector. **[3 Marks]**

iv). We let the following $p^{(n)} = (\alpha_3, \beta_3, \gamma_3)$ denote the distribution immediately before the interchange at time $t = n$. Given the initial Distribution $p^{(0)} = (1, 0, 0)$, find the probability of states, $p^{(3)} = (\alpha_3, \beta_3, \gamma_3)$, $p^{(4)} = (\alpha_4, \beta_4, \gamma_4)$ and prove that

$\sqrt{(\alpha_3 - \alpha_4)^2 + (\beta_3 - \beta_4)^2 + (\gamma_3 - \gamma_4)^2} < 0.07.$ **[6 Marks]**

QUESTION 4

a). The arrival rate of customer at a service window in a certain cinema hall follows a probability distribution with mean rate of 45 per hour. The service rate of the clerk follows Poisson distribution with mean of 60 per hour. [Assume a first come first served basis].

i). What is the probability of having no customers in the system? **[2 Marks]**

ii). What is the probability of having five customers in the system? **[1 Mark]**

iii). What is the average waiting time of a customer in the queue (W_q)?

[2 Marks]

iv). What is the average waiting time of an arrival that has to wait in the system (W_s)? **[1 Mark]**

v). Find the average queue length (L_q)? **[1 Mark]**

vi). Find the average number of customers in the queue (L_s)? **[1 Marks]**

vii). What is the variance of the Queue length? **[2 Marks]**

b). A manufacturing company has two production departments, designated as P1 and P2, and three service departments designates as S1, S2 and S3 and wishes to allocate all factory overhead to production department. A primary distribution of overhead to all departments has been made and is indicated below. The company makes secondary distribution of overhead from service departments to production departments on a reciprocal basis, recognizing the fact that services of one service of one service department are utilized by one another. Data regarding costs and allocations percentages are as follows;

Department	Direct cost Kshs.	Percentage allocation of total cost of the department		
		s_1	s_2	s_3
s_1	25000	0	30	10
s_2	124000	20	0	10
s_3	77000	10	10	0
p_1	120000	40	30	30
p_2	290000	30	30	50

From the table for example s_1 spends 20 % of its time and effort on s_2, 10 % on s_3, 40 % on p_1 and 30 % on p_2.

i). Find the transition matrix and re-arrange it in the form $P = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix}$. **[4 Marks]**

ii). What is the $\lim_{n\to\infty} P^n$, and the value of the fundamental matrix. **[6 Marks]**

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