MAASAI MARA UNIVERSITY

MAIN EXAMINATION 2018/2019 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER EXAMINATIONS

FOR

THE DEGREE OF BACHELOR OF SCIENCE

MAT 426: METHODS II

DATE: TIME:

DURATION: 2hrs

INSTRUCTIONS TO CANDIDATES

1. This paper contains **FOUR** (4) questions
2. Answer question **ONE (1)** and any other **TWO** (**2**) questions
3. Do not forget to write your Registration Number.

QUESTION ONE (30 MARKS)

a) Solve the following differential equation in terms of the Bessel's function
$xy'' - 3y' + xy = 0$ **(5mks)**

b) Let $\overline{A}^p = \frac{\partial \overline{x}^p}{\partial x^q} A^q$. Prove that $A^q = \frac{\partial x^q}{\partial \overline{x}^p} \overline{A}^p$ **(3mks)**

c) Show that the contraction of the outer product of the tensors A^p and B_q is an invariant. **(5mks)**

d) Use Neumann series method to solve the integral equation
$y(x) = 1 + \lambda \int_0^1 xzy(z)dz$ **(6mks)**

e) Prove the Rodrigues' Formula $P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l}(x^2 - 1)^l$ **(6mks)**

f) Show that $L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n}(x^n e^{-x})$ **(5mks)**

QUESTION TWO (20 MARKS)

a) Prove that the two independent solutions of Bessel's equation may be taken to be
$J_n(x)$ and $y_n(x) = \frac{\cos n\pi J_n(x) - J_{-n}(x)}{\sin n\pi}$ for all values of n, **(15mks)**

b) Prove that

$$\frac{\exp\left\{-xt/(1-t)\right\}}{(1-t)} = \sum_{n=0}^{\infty} L_n(x) t^n$$ **(5mks)**

QUESTION THREE (20 MARKS)

a) Given the generating function

$$\left(1-2xt+t^2\right)^{-\frac{1}{2}} = \sum_{t=0}^{\infty} t^l P_l(x)$$

Prove the recurrence relations

i. $lP_l(x) = (2l-1)xP_l(x) - (l-1)P_{l-1}(x) \qquad l \geq 2$ **(6mks)**

ii. $lP_l(x) = xP_l'(x) - P_{l-1}'(x)$ **(6mks)**

b) Find the eigen values and corresponding eigen functions of the following Fredholm equation

$$y(x) = \lambda \int_0^{\pi} \sin(x+z) y(z)\, dz$$ **(8mks)**

QUESTION FOUR (20 MARKS)

a) Use the Fredholm theory to show that the resolvent kernel to the integral equation $y(x) = x^{-3} + \lambda \int_a^b x^2 z^2 y(z)\, dz$ is $\dfrac{5x^2z^2}{5-\lambda\left(b^5-a^5\right)}$ and hence solve the equation. **(8mks)**

b) Show that the orthogonality properties of Laguerre polynomial is given by

$$\int_0^{\infty} e^{-x} L_n(x) L_m(x)\, dx = \delta_{n,m}.$$ **(8mks)**

c) Suppose A_r^{pq} and B_t^s are tensors. Prove that $C_{rt}^{pqs} = A_r^{pq} B_t^s$ is also a tensor. **(4mks)**